## Tab 1: Refinement in the Mathematics TEKS

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# Tab 1: Refinement in the Mathematics TEKS

## Master Materials List

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<th>Chart paper</th>
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### Algebra I, Algebra II, and Geometry TEKS
- 6-8 Mathematics TEKS
- K-12 Mathematics TEKS

### What Are the Changes 9-12? Transparencies and Handouts

The following materials are not in the notebook. They can be accessed on the CD through the links below.

- **Mathematics TEKS for Grades 6-8, Algebra 1, Geometry, and Algebra 2 on mailing labels**
- **Algebra 1, Algebra 2, and Geometry TEKS with a blank column for notes**
- **PowerPoint**
Activity: What Are the Changes 9-12?

TEKS: Mathematics 6th-8th grade TEKS and TEKS for Algebra I, Algebra II, and Geometry

Materials: Chart paper
- TEKS for Algebra 1, Algebra 2, and Geometry (page 1-8 – 1-18)
- Mathematics TEKS for Grades 6-8, Algebra 1, Geometry, and Algebra 2 on mailing labels, may be used instead of cutting the copies of the TEKS
- 6-8 Mathematics TEKS (page 1-19 – 1-29)
- K-12 Mathematics TEKS, one per group (page 1-30 - 72)
- Algebra I, Algebra II, and Geometry TEKS with a blank column for notes
- Transparencies 1 and 2 (pages 1-5, 1-7)
- Markers
- Highlighters
- PowerPoint slides
- Handout 1 (page 1-6)
- Significant Changes Guide 9-12 (page 1-73 – 1-86)
- Significant Changes Chart 9-12 (page 1-87 – 1-92)

Overview: Participants will investigate the refinements to the Texas Essential Knowledge and Skills and consider implications to instruction. They will also look at the refinements vertically to understand how instruction in each course and at preceding grade levels impact and complements each other. Special attention will be given to the implementation of new TEKS and the modifications that will need to take place to make sure all students receive instruction in the new concepts.

Grouping: Large group and small group

Time: 3 hours

Lesson:

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<td>1. Share PowerPoint slides that are relevant to changes in the 6-12 TEKS. (See the materials list for link to the PowerPoint.) Point out that this is just a sample of what will happen to the TEKS and the TAKS over the next two years and what needs to be taught for student success on TAKS.</td>
<td>This gives teachers a reason to want to do the alignment activity because it is just a sample of the critical changes to the TEKS and TAKS that will impact instruction in the classroom. What type of instructional strategies and questioning techniques must</td>
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<td>teachers use to address the spirit of the TEKS? What should the culture of the mathematics classroom look like and sound like?</td>
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<td>2. Sort participants by course: Algebra I, Algebra II, and Geometry. It is okay to have several groups working on the same course because one group may report back differently from the other.</td>
<td>If a teacher teaches more than one course, then he/she must choose one course for this activity. Make sure that you have at least one group for each course.</td>
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<td>3. Focus of each course group is:</td>
<td>Stress that it is really important that the participants not ignore the <strong>Basic Understandings</strong> for the assigned course.</td>
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<tr>
<td><strong>Algebra I:</strong> Consider the refinements in Algebra I TEKS and look back for connections in grades 6-8 TEKS and forward to connections in Algebra II.</td>
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<tr>
<td><strong>Geometry:</strong> Consider the refinements in Geometry TEKS and look back for connections in Algebra I and Grades 6-8 Geometry and Measurement Strands and forward to connections in Algebra II.</td>
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<td><strong>Algebra II:</strong> Consider the refinements in Algebra II TEKS and look back for connections in Geometry, Algebra I, and 8th Grade (if time permits) and forward to connections in PreCalculus.</td>
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<td>4. Determine where each group will work and have teachers move.</td>
<td>Have someone from each group pick up the TEKS that the group will need.</td>
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<tr>
<td>5. Direct each group to study the assigned content and starting with the refinements to the assigned course, look “forward” and “back” to the assigned grade level and course TEKS.</td>
<td>Use Transparency 1 (page 1-5) and make sure instructions are understood. Each group will have TEKS for the assigned course (pages 1-8 – 1-18) and the grade 6-8 TEKS (pages 1-19 – 1-30).</td>
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<td>Each group will make a chart showing:</td>
<td>Handout 1 (page 1-6) will help groups organize.</td>
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<tr>
<td>• the “new” or refined TEKS</td>
<td>Encourage participants to follow a concept back to where it is first</td>
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<td>• where the concept was introduced</td>
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<tr>
<td>• where the concept is mastered</td>
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<td>• any gaps. (These are places in the TEKS where a concept is addressed</td>
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<tr>
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<td>before the course and after the course but not in the course being studied.)</td>
<td>introduced.</td>
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<td>Because of the nature of the activity, some groups will finish before others. For those that finish early, have them look through the materials for English Language Learners in Tab 7. (Looking at these materials will be a working lunch assignment for everyone.)</td>
<td>Suggest that groups cut apart TEKS one course at a time. Also, since the K&amp;S statement may be cut apart from the SE, suggest that they label each SE with the K&amp;S number. (See the materials list for a link for the TEKS on peel and stick labels.)</td>
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<tr>
<td>A set of the K-12 TEKS (page 1-30 – 1-72) should also be provided so that participants can look to see if something deleted was actually moved to a different course or strand.</td>
<td><strong>If you are working with a group of novice teachers or teachers that have not been part of an alignment activity, you have the materials to do a complete alignment instead of just focusing on the refinements. This will take more time but will be very beneficial in the long run. Working with high school teachers, the complete alignment should be grades 6-12 at a minimum.</strong></td>
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<td>5. While in the course groups, participants should look at the changes in each course and identify the “most significant” changes by course.</td>
<td>Each person should record biggest changes for his/her particular course on the recording sheet provided.</td>
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<tr>
<td>6. Use Transparency 2 (page 1-7) to give instructions for the next part of the activity. Post the most significant changes as well as the vertical alignment of the changes around the room. Make sure all algebra 1 groups post their findings together. Do the same for Geometry and Algebra II groups.</td>
<td>The debriefing of this part of the activity should first involve a sharing among the course groups to compare their findings. Groups may need to make modifications to their findings and may even decide to make a combined chart from the work that they have done individually. A gallery walk will follow so each group needs to make sure that they are clearly presenting to other groups what they have learned about the changes in the</td>
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<td>TEKS for their course. Note: If something is deleted from the TEKS, the group needs to determine whether it has moved to a different course or if it is really deleted. Also, if something is added, is it really new or moved from another course? When debriefing this part of the activity, ask participants to stick to the changes instead of giving an overview of the course.</td>
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<td>7. Conduct a gallery walk so that all participants see the work completed for all three courses.</td>
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<td>8. Return to the large group and conduct a debriefing of the significant changes in the three courses as well as connections that have been discovered during the participants work.</td>
<td>A guide (pages 1-73 – 1-86) that highlights the significant changes and a chart (pages 1-87 – 1-92) with information about the type of changes and notes about those changes are provided for the Trainers to use. Copies of the TEKS on wide paper with the blank column provide a good place for participants to make notes about significant changes. (See the materials list for link to this version of the TEKS.)</td>
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</table>
Study your assigned course piece. Identify the “most significant” changes within the course.

On chart paper develop a vertical alignment for the changes in the assigned course part. Note any gaps or overlaps in the concepts. Trace each concept back to where it was first introduced. Be sure to note when a concept should be mastered.

If a concept has been deleted, is it no longer a part of the TEKS, or has it been moved to another course?

If a concept has been added to your strand, is it really new or has it been moved from another course?
| New TEKS Statement | Nature of Change (New concept, deleted concept, clarify language, etc.) | First Introduced | Should be mastered | Implications for the Classroom |
Record “most significant” changes for your course on chart paper.

Post
Chapter 111. Texas Essential Knowledge and Skills for Mathematics
Subchapter C. High School

§111.31. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades 9-12.
The provisions of this subchapter shall be implemented beginning September 1, 1998, and at that
time, shall supersede §75.63(e)-(g) of this title (relating to Mathematics).

§111.32. Algebra I (One Credit).
(a) Basic understandings.
   (1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic
   understandings of number, operation, and quantitative reasoning; patterns, relationships, and
   algebraic thinking; geometry; measurement; and probability and statistics are essential
   foundations for all work in high school mathematics. Students will continue to build on this
   foundation as they expand their understanding through other mathematical experiences.
   (2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra;
symbols provide powerful ways to represent mathematical situations and to express
   generalizations. Students use symbols in a variety of ways to study relationships among
   quantities.
   (3) Function concepts. [Functions represent the systematic dependence of one quantity on another.] A
   function is a fundamental mathematical concept; it expresses a special kind of relationship
   between two quantities. Students use functions to determine one quantity from another, to
   represent and model problem situations, and to analyze and interpret relationships.
   (4) Relationship between equations and functions. Equations and inequalities arise as a way of asking
   and answering questions involving functional relationships. Students work in many situations to set
   up equations and inequalities and use a variety of methods to solve them. These equations.
   (5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in
   understanding underlying relationships. Students use a variety of representations (concrete,
pictorial, numerical, symbolic, algorithmic, graphical, and verbal), tools, and technology (including,
but not limited to, powerful and accessible hand-held calculators with graphing
   capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.
   (6) Underlying mathematical processes. Many processes underlie all content areas in mathematics.
   As they do mathematics, students continually use problem-solving, computation in problem-
solving contexts, language and communication, and reasoning (justification and proof) to make
   connections within and outside mathematics. Students also use and reasoning, as well as
   multiple representations, technology, applications and modeling, and numerical fluency in
   problem-solving contexts (justification and proof).

(b) Knowledge and skills
   (A.1) Foundations for functions. The student understands that a function represents a dependence
   of one quantity on another and can be described in a variety of ways.

The student is expected to:
   (A) describe[s] independent and dependent quantities in
   functional relationships.
   (B) gather[s] and record[s] data and use [or uses] data
   sets to determine functional relationships between quantities.
   (C) describe[s] functional relationships for given problem
   situations and write [writes] equations or inequalities to
   answer questions arising from the situations.
   (D) represent[s] relationships among quantities using
   concrete models, tables, graphs, diagrams, verbal
   descriptions, equations, and inequalities; and
   (E) interpret and make decisions, predictions, and critical
   judgments from functional relationships.
(A.2) **Foundations for functions.** The student uses the properties and attributes of functions.

The student is expected to:

(A) identify[s] and sketch[s] the general forms of linear \((y = x)\) and quadratic \((y = x^2)\) parent functions;

(B) identify[s] the mathematical domains and ranges and determine [determines] reasonable domain and range values for given situations, both continuous and discrete;

(C) interpret[s] situations in terms of given graphs or creates situations that fit given graphs; and

(D) collect[s] and organize[s] data, make and interpret [makes and interprets] scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make [models, predicts, and makes] decisions and critical judgments in problem situations.

(A.3) **Foundations for functions.** The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use[s] symbols to represent unknowns and variables; and

(B) look[s] for patterns and represent [represents] generalizations algebraically.

(A.4) **Foundations for functions.** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find[s] specific function values, simplify [simplifies] polynomial expressions, transform and solve [transforms and solves] equations, and factor [factors] as necessary in problem situations;

(B) use[s] the commutative, associative, and distributive properties to simplify algebraic expressions;

(C) connect equation notation with function notation, such as \(y = x + 1\) and \(f(x) = x + 1\).

(A.5) **Linear functions.** The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(A) determine[s] whether or not given situations can be represented by linear functions;

(B) determine[s] the domain and range [values] for [which] linear functions in [make sense for] given situations; and

(C) use[s], translate[s], and make connections [The student translates] among [and uses] algebraic, tabular, graphical, or verbal descriptions of linear functions.

(A.6) **Linear functions.** The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(A) develop[s] the concept of slope as rate of change and determine [determines] slopes from graphs, tables, and algebraic representations;

(B) interpret[s] the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigate[s], describe[s], and predict[s] the effects of changes in \(m\) and \(b\) on the graph of \(y = mx + b\);

(D) graph[s] and write[s] equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept.
(E) determine[s] the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representations ; [•]
(F) interpret[s] and predict[s] the effects of changing slope and y-intercept in applied situations ; and [•]
(G) relate[s] direct variation to linear functions and solve [solves] problems involving proportional change.

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:
(A) analyze[s] situations involving linear functions and formulate [formulates] linear equations or inequalities to solve problems ; [•]
(B) investigate[s] methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select [selects] a method, and solve [solves] the equations and inequalities ; and [•]
(C) interpret[s] and determine[s] the reasonableness of solutions to linear equations and inequalities.

(A.8) Linear functions. The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:
(A) analyze[s] [The student analyzes] situations and formulate [formulates] systems of linear equations in two unknowns to solve problems ; [•]
(B) solve[s] [The student solves] systems of linear equations using concrete models, graphs, tables, and algebraic methods ; and [•]
(C) interpret[s] and determine[s] [For given contexts, the student interprets and determines] the reasonableness of solutions to systems of linear equations.

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:
(A) determine[s] the domain and range [values] for [which] quadratic functions in [make sense for] given situations ; [•]
(B) investigate[s], describe[s], and predict[s] the effects of changes in a on the graph of \( y = ax^2 + c \); \( y = ax^2 \) .
(C) investigate[s], describe[s], and predict[s] the effects of changes in c on the graph of \( y = ax^2 + c \); and \( y = x^2 + c \).
(D) analyze[s] graphs of quadratic functions and draw [draws] conclusions.

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:
(A) solve[s] quadratic equations using concrete models, tables, graphs, and algebraic methods ; and [•]
(B) make connections among the solutions (roots) [The student relates the solutions] of quadratic equations , the zeros of their related functions, and the horizontal intercepts (x-intercepts) of the graph [of the roots] of the function [their functions].

(A.11) Quadratic and other nonlinear functions. The student understands there are situations modeled by functions that are neither linear nor quadratic and

The student is expected to:
(A) use[s] patterns to generate the laws of exponents and apply [applies] them in problem-solving situations ; [•]
(B) analyze[s] data and represent [represents] situations involving inverse variation using concrete models,
models the situations. tables, graphs, or algebraic methods; and [ ]

(C) analyze[s] data and represent [represents] situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.
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time, shall supersede §75.63(e)-(g) of this title (relating to Mathematics).

§111.33. Algebra II (One-Half to One Credit).
(a) Basic understandings.
   (1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic
   understandings of number, operation, and quantitative reasoning; patterns, relationships, and
   algebraic thinking; geometry; measurement; and probability and statistics are essential
   foundations for all work in high school mathematics. Students continue to build on this foundation
   as they expand their understanding through other mathematical experiences.
   (2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra;
symbols provide powerful ways to represent mathematical situations and to express
generalizations. Students study algebraic concepts and the relationships among them to better
understand the structure of algebra.
   (3) Functions, equations, and their relationship. The study of functions, equations, and their
   relationship is central to all of mathematics. Students perceive functions and equations as means
for analyzing and understanding a broad variety of relationships and as a useful tool for
expressing generalizations.
   (4) Relationship between algebra and geometry. Equations and functions are algebraic tools that can
be used to represent geometric curves and figures; similarly, geometric figures can illustrate
algebraic relationships. Students perceive the connections between algebra and geometry and
use the tools of one to help solve problems in the other.
   (5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in
understanding underlying relationships. Students use a variety of representations (concrete,
pictorial, numerical, symbolic, [algorithmic], graphical, and verbal), tools, and technology [including, but not limited to, powerful and accessible hand-held] calculators with graphing
capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.
   (6) Underlying mathematical processes. Many processes underlie all content areas in mathematics.
As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make
connections within and outside mathematics. Students also use [and reasoning, as well as multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.]

(b) Knowledge and skills.
(2A.1) Foundations for functions. The student uses properties and attributes of functions and applies functions to problem situations.
The student is expected to
(A) identify[s] [For a variety of situations, the student identifies] the mathematical domains and ranges of
functions and determine [determines] reasonable domain and range values for continuous and discrete
[given] situations; and [it]
(B) collect[s] and organize[s] [In solving problems, the student collects data and records results, organizes the]
data, make and interpret [makes] scatterplots, fit [fits] the graph of a [curves to the appropriate parent] function to
the data, interpret [interprets] the results, and proceed [proceeds] to model, predict, and make decisions and
critical judgments.
(2A.2) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:
(A) use[s] tools including matrices, factoring, and properties of exponents to simplify expressions and to transform and solve equations; and
(B) use[s] complex numbers to describe the solutions of quadratic equations.

(C) The student connects the function notation of \( y = f(x) \) and \( f(x) = y \).

(2A.3) Foundations for functions. The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations.

The student is expected to:
(A) analyze[s] situations and formulate systems of equations in two or more unknowns or inequalities in two or more unknowns to solve problems;
(B) use[s] algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities;
(C) interpret[s] and determine[s] the reasonableness of solutions to systems of equations or inequalities for given contexts.

(2A.4) Algebra and geometry. The student connects algebraic and geometric representations of functions.

The student is expected to:
(A) identify[s] and sketch[s] graphs of parent functions, including linear \( f(x) = x \) \( (y = x) \), quadratic \( f(x) = x^2 \) \( (y = x^2) \), square root \( y = \sqrt{x} \), inverse \( y = \frac{1}{x} \), exponential \( f(x) = ax \) \( (y = ax) \), and logarithmic \( f(x) = \log_{ax} \) functions, absolute value \( f(x) = |x| \), square root of \( x \) \( (f(x) = \sqrt{x}) \), and reciprocal of \( x \) \( (f(x) = \frac{1}{x}) \);
(B) extend[s] parent functions with parameters such as \( a \) in \( f(x) = ax \) \( (y = mx) \) and describe the effects of the parameter changes on the graph of parent functions; and
(C) describe and analyze the relationship between a function and its inverse. The student recognizes inverse relationships between various functions.

(2A.5) Algebra and geometry. The student knows the relationship between the geometric and algebraic descriptions of conic sections.

The student is expected to:
(A) describe[s] a conic section as the intersection of a plane and a cone;
(B) sketch graphs of conic sections to relate[s] simple parameter changes in the equation to corresponding changes in the graph;
(C) identify[s] symmetries from graphs of conic sections;
(D) identify[s] the conic section from a given equation; and
(E) use[s] the method of completing the square.

(2A.6) Quadratic and square root functions. The student understands that quadratic functions can be represented in different ways and translates among their various representations.

The student is expected to:
(A) determine[s] the reasonable domain and range values of quadratic functions, as well as interpret[s] and determine[s] the reasonableness of solutions to quadratic equations and inequalities;
(B) relate[s] representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions;
(2A.7) Quadratic and square root functions. The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations.

The student is expected to:
(A) use characteristics of the quadratic parent function to sketch the related graphs and connect between the \( y = ax^2 + bx + c \) and the \( y = a(x - h)^2 + k \) symbolic representations of quadratic functions; and
(B) use the parent function to investigate, describe, and predict the effects of changes in \( a, h, \) and \( k \) on the graphs of \( y = a(x - h)^2 + k \) form of a function in applied and purely mathematical situations.

(2A.8) Quadratic and square root functions. The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:
(A) analyze situations involving quadratic functions and formulate quadratic equations or inequalities to solve problems;
(B) analyze and interpret the solutions of quadratic equations using discriminants and solve quadratic equations using the quadratic formula;
(C) compare and translate between algebraic and graphical solutions of quadratic equations; and
(D) solve quadratic equations and inequalities using graphs, tables, and algebraic methods.

(2A.9) Quadratic and square root functions. The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:
(A) use the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describe limitations on the domains and ranges;
(B) relate representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions;
(C) determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities;
(D) determine solutions of square root equations and inequalities using graphs, tables, and algebraic methods;
(E) determine solutions of square root inequalities using graphs and tables;
(F) analyze situations modeled by square root functions, formulate equations or inequalities, select a method, and solve problems; and
(G) connect inverses of square root functions with quadratic functions.

(2A.10) Rational functions. The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and

The student is expected to:
(A) use quotients of polynomials to describe the graphs of rational functions, predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic
analyzes the solutions in terms of the situation.

behavior ; [1]

(B) analyze[s] various representations of rational functions with respect to problem situations ; [1]

(C) determine[s] the reasonable domain and range values of rational functions, as well as interpret and determine [interprets and determines] the reasonableness of solutions to rational equations and inequalities ; [1]

(D) determine the solutions of [The student solves] rational equations [and inequalities] using graphs, tables, and algebraic methods ; [1]

(E) determine solutions of rational inequalities using graphs and tables;

(F) analyze[s] [The student analyzes] a situation modeled by a rational function, formulate [formulates] an equation or inequality composed of a linear or quadratic function, and solve [solves] the problem ; and [1]

(G) determine solutions of rational inequalities using graphs and tables;

(2A.11) Exponential and logarithmic functions. The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) develop[s] [the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses ; [1]

(B) use[s] the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe [describes] limitations on the domains and ranges, and examine [examines] asymptotic behavior ; [1]

(C) determine[s] the reasonable domain and range values of exponential and logarithmic functions, as well as interpret and determine [interprets and determines] the reasonableness of solutions to exponential and logarithmic equations and inequalities ; [1]

(D) determine solutions of [The student solves] exponential and logarithmic equations [and inequalities] using graphs, tables, and algebraic methods ; [1]

(E) determine solutions of exponential and logarithmic inequalities using graphs and tables; and

(F) analyze[s] a situation modeled by an exponential function, formulate [formulates] an equation or inequality, and solve [solves] the problem.
Chapter 111. Texas Essential Knowledge and Skills for Mathematics
Subchapter C. High School

§111.31. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades 9-12.
The provisions of this subchapter shall be implemented beginning September 1, 1998, and at that
time, shall supersede §75.63(e)-(g) of this title (relating to Mathematics).

§111.34. Geometry (One Credit).
(a) Basic understandings.
   (1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic
       understandings of number, operation, and quantitative reasoning; patterns, relationships, and
       algebraic thinking; geometry; measurement; and probability and statistics are essential
       foundations for all work in high school mathematics. Students continue to build on this foundation
       as they expand their understanding through other mathematical experiences.
   (2) Geometric thinking and spatial reasoning. Spatial reasoning plays a critical role in geometry;
       geometric shapes and figures provide powerful ways to represent mathematical situations and
       to express generalizations about space and spatial relationships. Students use geometric thinking
       to understand mathematical concepts and the relationships among them.
   (3) Geometric figures and their properties. Geometry consists of the study of geometric figures of
       zero, one, two, and three dimensions and the relationships among them. Students study
       properties and relationships having to do with size, shape, location, direction, and orientation of
       these figures.
   (4) The relationship between geometry, other mathematics, and other disciplines. Geometry can be
       used to model and represent many mathematical and real-world situations. Students perceive the
       connection between geometry and the real and mathematical worlds and use geometric ideas,
       relationships, and properties to solve problems.
   (5) Tools for geometric thinking. Techniques for working with spatial figures and their properties
       are essential in understanding underlying relationships. Students use a variety of representations
       (concrete, pictorial, numerical, symbolic, graphical, and verbal [algebraic and coordinate]), tools,
       and technology [including, but not limited to, powerful and accessible hand-held]
       calculators with graphing capabilities, data collection devices, and computers
       [with graphing capabilities] to solve meaningful problems by representing and transforming figures
       [transforming figures] and analyzing relationships [and proving things about them].
   (6) Underlying mathematical processes. Many processes underlie all content areas in mathematics.
       As they do mathematics, students continually use problem-solving, [computation in problem-
       solving contexts,] language and communication, connections within and outside mathematics,
       and reasoning (justification and proof). Students also use [as well as] multiple representations,
       technology, applications and modeling, and numerical fluency in problem solving contexts
       [justification and proof].

(b) Knowledge and skills.
   (G.1) Geometric structure. The student understands the structure of, and relationships within, an axiomatic
       system.
       The student is expected to:
       (A) develop[s] an awareness of the structure of a
           mathematical system, connecting definitions, postulates,
           logical reasoning, and theorems;
       (B) recognize[s] [Through] the historical development of
           geometric systems[the student recognizes that] and
           know mathematics is developed for a variety of purposes
           (C) compare[s] and contrast[s] the structures and
               implications of Euclidean and non-Euclidean geometries.

   (G.2) Geometric structure. The student analyzes geometric relationships
       in order to make and verify conjectures
       The student is expected to:
       (A) use[s] constructions to explore attributes of geometric
           figures and to make conjectures about geometric
           relationships; and
       (B) make[s] [The student makes and verifies] conjectures
           about angles, lines, polygons, circles, and three-
dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

The student is expected to:

(A) determine[s] the validity [The student determines if the converse of a conditional statement, its converse, inverse, and contrapositive; [is true or false.

(B) construct[s] and justify[s] statements about geometric figures and their properties ; []

(C) use logical reasoning [The student demonstrates what it means] to prove statements are true and find counter examples to disprove [mathematically that] statements that are false; [ true.

(D) use[s] inductive reasoning to formulate a conjecture; and

(E) use[s] deductive reasoning to prove a statement.

The student is expected to select[s] an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.

The student is expected to:

(A) use[s] numeric and geometric patterns to develop algebraic expressions representing geometric properties; to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles;

(B) use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles;

(C) use[s] properties of transformations and their compositions to make connections between mathematics and the real world, [in applications] such as tessellations; and [or fractals.]

(D) identify[s] and apply[s] patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.

The student is expected to:

(A) describe and draw the intersection of a given plane with various [The student describes, and draws cross sections and other slices of] three-dimensional geometric figures; [objects,]

(B) use[s] [The student uses] nets to represent and construct three- dimensional geometric figures; and [objects,]

(C) use orthographic and isometric views [The student uses top, front, side, and corner views] of three-dimensional geometric figures [objects] to represent and construct three-dimensional geometric figures [create accurate and complete representations] and solve problems.
(G.7) Dimensionality and the geometry of location. The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly.

The student is expected to:
(A) use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures;
(B) use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons; and
(C) derive and use formulas involving length, slope, including distance and midpoint.

(G.8) Congruence and the geometry of size. The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, [perimeter] and volume in problem situations.

The student is expected to:
(A) find areas of regular polygons, circles, and composite figures;
(B) find areas of sectors and arc lengths of circles using proportional reasoning;
(C) derive, extend, and use the Pythagorean Theorem; and
(D) find surface areas and volumes of prisms, pyramids, spheres, cones, and cylinders, and composites of these figures in problem situations.

(G.9) Congruence and the geometry of size. The student analyzes properties and describes relationships in geometric figures.

The student is expected to:
(A) formulate and test conjectures about the properties of parallel and perpendicular lines based on explorations and concrete models;
(B) formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models;
(C) formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models; and
(D) analyze the characteristics of polyhedra and other three-dimensional figures and their component parts based on explorations and concrete models.

(G.10) Congruence and the geometry of size. The student applies the concept of congruence to justify properties of figures and solve problems.

The student is expected to:
(A) use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane; and
(B) justify and apply triangle congruence relationships.

(G.11) Similarity and the geometry of shape. The student applies the concepts of similarity to justify properties of figures and solve problems.

The student is expected to:
(A) use and extend similarity properties and transformations to explore and justify conjectures about geometric figures;
(B) use ratios to solve problems involving similar figures;
(C) develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods; and
(D) describe the effect on perimeter, area, and volume when one or more dimensions [length, width, or height] of a figure are changed and apply this idea in solving problems.
Chapter 111. Texas Essential Knowledge and Skills for Mathematics
Subchapter A. Elementary (6-8)

§111.11. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades K-5.
The provisions of this subchapter shall be implemented by school districts beginning with the 2006-2007 school year. [September 1, 1998, and at that time shall supersede §75.27(a)-(f) of this title (relating to Mathematics)]

§111.22. Mathematics, Grade 6.

(a) Introduction.
(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 6 are using ratios to describe direct proportional relationships involving number, geometry, measurement, and probability, and adding and subtracting decimals and fractions.

(2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures (objects) or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.

(3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology (at least four-function calculators for whole numbers, decimals, and fractions) and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.

(b) Knowledge and skills. (6.1)
Number, operation, and quantitative reasoning. The student represents and uses rational numbers in a variety of equivalent forms.

The student is expected to:
(A) compare and order non-negative rational numbers;
(B) generate equivalent forms of rational numbers including whole numbers, fractions, and decimals;
(C) use integers to represent real-life situations;
(D) write prime factorizations using exponents; [and]
(E) identify factors of a positive integer, [and multiples including] common factors, and the greatest common factor of a set of positive integers; [and] common multiples.
(F) identify multiples of a positive integer and common multiples and the least common multiple of a set of positive integers.

(6.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, and divides to solve problems and justify solutions.

The student is expected to:
(A) model addition and subtraction situations involving fractions with objects, pictures, words, and numbers; [and]
(B) use addition and subtraction to solve problems involving fractions and decimals;
(C) use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates; [and]
(6.3) **Patterns, relationships, and algebraic thinking.** The student solves problems involving direct proportional relationships.

The student is expected to:

(A) use ratios to describe proportional situations;

(B) represent ratios and percents with concrete models, fractions, and decimals; and

(C) use ratios to make predictions in proportional situations.

(6.4) **Patterns, relationships, and algebraic thinking.** The student uses letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes.

The student is expected to:

(A) use tables and symbols to represent and describe proportional and other relationships such as those involving conversions, arithmetic sequences (with a constant rate of change), perimeter and area [etc.]; and

(B) use tables of data to generate formulas representing [to represent] relationships involving perimeter, area, volume of a rectangular prism, etc. [from a table of data].

(6.5) **Patterns, relationships, and algebraic thinking.** The student uses letters to represent an unknown in an equation.

The student is expected to formulate equations [an equation] from [a] problem situations described by linear relationships [situation].

(6.6) **Geometry and spatial reasoning.** The student uses geometric vocabulary to describe angles, polygons, and circles.

The student is expected to:

(A) use angle measurements to classify angles as acute, obtuse, or right;

(B) identify relationships involving angles in triangles and quadrilaterals; and

(C) describe the relationship between radius, diameter, and circumference of a circle.

(6.7) **Geometry and spatial reasoning.** The student uses coordinate geometry to identify location in two dimensions.

The student is expected to locate and name points on a coordinate plane using ordered pairs of non-negative rational numbers.

(6.8) **Measurement.** The student solves application problems involving estimation and measurement of length, area, time, temperature, volume [capacity], weight, and angles.

The student is expected to:

(A) estimate measurements (including circumference) and evaluate reasonableness of results;

(B) select and use appropriate units, tools, or formulas to measure and to solve problems involving length (including perimeter [and circumference]), area, time, temperature, volume [capacity], and weight;

(C) measure angles; and

(D) convert measures within the same measurement system (customary and metric) based on relationships between units.
(6.9) **Probability and statistics.** The student uses experimental and theoretical probability to make predictions. The student is expected to:
(A) construct sample spaces using lists and tree diagrams; and
(B) find the probabilities of a simple event and its complement and describe the relationship between the two.

(6.10) **Probability and statistics.** The student uses statistical representations to analyze data. The student is expected to:
(A) select and use an appropriate representation for presenting and displaying different graphical representations of the same data including line plot, line graph, bar graph, and stem and leaf plot;
(B) identify mean (using concrete objects and pictorial models), median, mode, and range of a set of data; and
(C) sketch circle graphs to display data; and
(D) solve problems by collecting, organizing, displaying, and interpreting data.

(6.11) **Underlying processes and mathematical tools.** The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:
(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;
(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.

(6.12) **Underlying processes and mathematical tools.** The student communicates about Grade 6 mathematics through informal and mathematical language, representations, and models. The student is expected to:
(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and
(B) evaluate the effectiveness of different representations to communicate ideas.

(6.13) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:
(A) make conjectures from patterns or sets of examples and non-examples; and
(B) validate his/her conclusions using mathematical properties and relationships.
§111.23. Mathematics, Grade 7.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 7 are using direct proportional relationships in number, geometry, measurement, and probability; applying addition, subtraction, multiplication, and division of decimals, fractions, and integers; and using statistical measures to describe data.

(2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures (objects) or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.

(3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology ([at least four-function calculators for whole numbers, decimals, and fractions]) and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.

(b) Knowledge and skills.

(7.1) **Number, operation, and quantitative reasoning.** The student represents and uses numbers in a variety of equivalent forms.

The student is expected to:

(A) compare and order integers and positive rational numbers;

(B) convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator; and

(C) represent squares and square roots using geometric models.

(7.2) **Number, operation, and quantitative reasoning.** The student adds, subtracts, multiplies, or divides to solve problems and justify solutions.

The student is expected to:

(A) represent multiplication and division situations involving fractions and decimals with [concrete] models, including concrete objects, pictures, words, and numbers;

(B) use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals;

(C) use models, such as concrete objects, pictorial models, and number lines, to add, subtract, multiply, and divide integers and connect the actions to algorithms;

(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio;

(E) simplify numerical expressions involving order of operations and exponents;

(F) select and use appropriate operations to solve problems and justify the selections; and

(G) determine the reasonableness of a solution to a problem.
(7.3) **Patterns, relationships, and algebraic thinking.** The student solves problems involving direct proportional relationships. The student is expected to:
(A) estimate and find solutions to application problems involving percent; and
(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.

(7.4) **Patterns, relationships, and algebraic thinking.** The student represents a relationship in numerical, geometric, verbal, and symbolic form. The student is expected to:
(A) generate formulas involving unit conversions, perimeter, area, circumference, volume, and scaling;
(B) graph data to demonstrate relationships in familiar concepts such as conversions, perimeter, area, circumference, volume, and scaling; and
(C) use words and symbols to describe the relationship between the terms in an arithmetic sequence (with a constant rate of change) and their positions in the sequence.

(7.5) **Patterns, relationships, and algebraic thinking.** The student uses equations to solve problems. The student is expected to:
(A) use concrete and pictorial models to solve equations and use symbols to record the actions; and
(B) formulate a possible problem situation when given a simple equation and formulate an equation when given a problem situation.

(7.6) **Geometry and spatial reasoning.** The student compares and classifies two- and three-dimensional figures using geometric vocabulary and properties. The student is expected to:
(A) use angle measurements to classify pairs of angles as complementary or supplementary;
(B) use properties to classify shapes including triangles and quadrilaterals [pentagons, and circles]; and
(C) use properties to classify three-dimensional figures, including pyramids, cones, prisms, and cylinders; and
(D) use critical attributes to define similarity.

(7.7) **Geometry and spatial reasoning.** The student uses coordinate geometry to describe location on a plane. The student is expected to:
(A) locate and name points on a coordinate plane using ordered pairs of integers; and
(B) graph reflections across the horizontal or vertical axis and graph translations on a coordinate plane.

(7.8) **Geometry and spatial reasoning.** The student uses geometry to model and describe the physical world. The student is expected to:
(A) sketch three-dimensional figures when given the top, side, and front views;
(B) make a net (two-dimensional model) of the surface area of a three-dimensional figure; and
(C) use geometric concepts and properties to solve problems in fields such as art and architecture.

(7.9) **Measurement.** The student solves application problems involving estimation and measurement. The student is expected to:
(A) estimate measurements and solve application problems involving length (including perimeter and circumference), area, and volume.
(7.10) **Probability and statistics.** The student recognizes that a physical or mathematical model can be used to describe the experimental and theoretical probability of real-life events.

The student is expected to:
(A) construct sample spaces for simple or composite experiments [compound events (dependent and independent)] ; and
(B) find the [approximate] probability of independent events [a compound event through experimentation].

(7.11) **Probability and statistics.** The student understands that the way a set of data is displayed influences its interpretation.

The student is expected to:
(A) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plot, line graph, bar graph, stem and leaf plot, circle graph, and Venn diagrams, and justify the selection; and
(B) make inferences and convincing arguments based on an analysis of given or collected data.

(7.12) **Probability and statistics.** The student uses measures of central tendency and range to describe a set of data.

The student is expected to:
(A) describe a set of data using mean, median, mode, and range; and
(B) choose among mean, median, mode, or range to describe a set of data and justify the choice for a particular situation.

(7.13) **Underlying processes and mathematical tools.** The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school.

The student is expected to:
(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;
(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
(D) select tools such as real objects, manipulatives, paper-pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.

(7.14) **Underlying processes and mathematical tools.** The student communicates about Grade 7 mathematics through informal and mathematical language, representations, and models.

The student is expected to:
(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and
(B) evaluate the effectiveness of different representations to communicate ideas.
(7.15) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:

(A) make conjectures from patterns or sets of examples and non-examples; and

(B) validate his/her conclusions using mathematical properties and relationships.
§111.24. Mathematics, Grade 8.
(a) Introduction.
   (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 8 are using basic principles of algebra to analyze and represent both proportional and non-proportional linear relationships and using probability to describe data and make predictions.
   (2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures [objects] or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.
   (3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology [(at least four function calculators for whole numbers, decimals, and fractions)] and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.

(b) Knowledge and skills.
(8.1) **Number, operation, and quantitative reasoning.** The student understands that different forms of numbers are appropriate for different situations.
   The student is expected to:
   (A) compare and order rational numbers in various forms including integers, percents, and positive and negative fractions and decimals;
   (B) select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships;
   (C) approximate (mentally and with calculators) the value of irrational numbers as they arise from problem situations (such as π, √2); and
   (D) express numbers in scientific notation, including negative exponents, in appropriate problem situations [using a calculator].

(8.2) **Number, operation, and quantitative reasoning.** The student selects and uses appropriate operations to solve problems and justify solutions.
   The student is expected to:
   (A) select [and use] appropriate operations to solve problems involving rational numbers and justify the selections;
   (B) use appropriate operations to solve problems involving [add, subtract, multiply, and divide] rational numbers in problem situations;
   (C) evaluate a solution for reasonableness; and
   (D) use multiplication by a constant factor (unit rate) to represent proportional relationships [for example, the arm span of a gibbon is about 1.4 times its height, a = 1.4h].

(8.3) **Patterns, relationships, and algebraic thinking.** The student identifies proportional or non-
   The student is expected to:
   (A) compare and contrast proportional and non-proportional linear relationships; and
(B) estimate and find solutions to application problems involving percents and other proportional relationships such as similarity and rates.

(8.4) **Patterns, relationships, and algebraic thinking.** The student makes connections among various representations of a numerical relationship.

The student is expected to generate a different representation of data given another representation of data (such as a table, graph, equation, or verbal description).

(8.5) **Patterns, relationships, and algebraic thinking.** The student uses graphs, tables, and algebraic representations to make predictions and solve problems.

The student is expected to:

(A) predict, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations; and

(B) find and evaluate an algebraic expression to determine any term in an arithmetic sequence (with a constant rate of change).

(8.6) **Geometry and spatial reasoning.** The student uses transformational geometry to develop spatial sense.

The student is expected to:

(A) generate similar figures using dilations including enlargements and reductions; and

(B) graph dilations, reflections, and translations on a coordinate plane.

(8.7) **Geometry and spatial reasoning.** The student uses geometry to model and describe the physical world.

The student is expected to:

(A) draw three-dimensional figures from different perspectives;

(B) use geometric concepts and properties to solve problems in fields such as art and architecture;

(C) use pictures or models to demonstrate the Pythagorean Theorem; and

(D) locate and name points on a coordinate plane using ordered pairs of rational numbers.

(8.8) **Measurement.** The student uses procedures to determine measures of three-dimensional figures.

The student is expected to:

(A) find lateral and total surface area of prisms, pyramids, and cylinders using concrete models and nets (two-dimensional models);

(B) connect models to formulas of prisms, cylinders, pyramids, spheres, and cones to formulas for volume of these objects; and

(C) estimate measurements and use formulas to solve application problems involving lateral and total surface area and volume.

(8.9) **Measurement.** The student uses indirect measurement to solve problems.

The student is expected to:

(A) use the Pythagorean Theorem to solve real-life problems; and

(B) use proportional relationships in similar two-dimensional figures or similar three-dimensional figures to find missing measurements.
<table>
<thead>
<tr>
<th>TEKS Code</th>
<th>Standard Description</th>
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<tbody>
<tr>
<td>(8.10) Measurement.</td>
<td>The student describes how changes in dimensions affect linear, area, and volume measures. &lt;br&gt; The student is expected to: &lt;br&gt; (A) describe the resulting effects on perimeter and area when dimensions of a shape are changed proportionally; and &lt;br&gt; (B) describe the resulting effect on volume when dimensions of a solid are changed proportionally.</td>
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<tr>
<td>(8.11) Probability and statistics.</td>
<td>The student applies concepts of theoretical and experimental probability to make predictions. &lt;br&gt; The student is expected to: &lt;br&gt; (A) find the probabilities of dependent and independent [compound] events [(dependent and independent)]; &lt;br&gt; (B) use theoretical probabilities and experimental results to make predictions and decisions; and &lt;br&gt; (C) select and use different models to simulate an event.</td>
</tr>
<tr>
<td>(8.12) Probability and statistics.</td>
<td>The student uses statistical procedures to describe data. &lt;br&gt; The student is expected to: &lt;br&gt; (A) select the appropriate measure of central tendency or range to describe a set of data and justify the choice for a particular situation [purpose]; &lt;br&gt; (B) draw conclusions and make predictions by analyzing trends in scatterplots; and &lt;br&gt; (C) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plots, line graphs, stem and leaf plots, [construct] circle graphs, bar graphs, box and whisker plots, [and] histograms, and Venn diagrams, with and without the use of technology.</td>
</tr>
<tr>
<td>(8.13) Probability and statistics.</td>
<td>The student evaluates predictions and conclusions based on statistical data. &lt;br&gt; The student is expected to: &lt;br&gt; (A) evaluate methods of sampling to determine validity of an inference made from a set of data; and &lt;br&gt; (B) recognize misuses of graphical or numerical information and evaluate predictions and conclusions based on data analysis.</td>
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<tr>
<td>(8.14) Underlying processes and mathematical tools.</td>
<td>The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. &lt;br&gt; The student is expected to: &lt;br&gt; (A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics; &lt;br&gt; (B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness; &lt;br&gt; (C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and &lt;br&gt; (D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.</td>
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</table>
(8.15) **Underlying processes and mathematical tools.** The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models.

The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and

(B) evaluate the effectiveness of different representations to communicate ideas.

(8.16) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions.

The student is expected to:

(A) make conjectures from patterns or sets of examples and nonexamples; and

(B) validate his/her conclusions using mathematical properties and relationships.
Chapter 111. Texas Essential Knowledge and Skills for Mathematics
Subchapter A. Elementary

§111.11. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades K-5.
The provisions of this subchapter shall be implemented by school districts beginning with the 2006-2007 school year. [September 1, 1998, and at that time shall supersede §75.27(a)-(f) of this title (relating to Mathematics)]

(a) Introduction.
(1) Within a well-balanced mathematics curriculum, the primary focal points at Kindergarten are developing whole-number concepts and using patterns and sorting to explore number, data, and shape.
(2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use patterns to describe objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal use of geometric properties to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.
(3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.
(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.
(K.1) Number, operation, and quantitative reasoning. The student uses numbers to name quantities.
The student is expected to:
(A) use one-to-one correspondence and language such as more than, same number as, or two less than to describe relative sizes of sets of concrete objects;
(B) use sets of concrete objects to represent quantities given in verbal or written form (through 20); and
(C) use numbers to describe how many objects are in a set (through 20) using verbal and symbolic descriptions.

(K.2) Number, operation, and quantitative reasoning. The student describes order of events or objects.
The student is expected to:
(A) use language such as before or after to describe relative position in a sequence of events or objects; and
(B) name the ordinal positions in a sequence such as first, second, third, etc.
(K.3) **Number, operation, and quantitative reasoning.** The student recognizes that there are quantities less than a whole. The student is expected to:
(A) share a whole by separating it into two equal parts; and
(B) explain why a given part is half of the whole.

(K.4) **Number, operation, and quantitative reasoning.** The student models addition (joining) and subtraction (separating). The student is expected to model and create addition and subtraction problems in real situations with concrete objects.

(K.5) **Patterns, relationships, and algebraic thinking.** The student identifies, extends, and creates patterns. The student is expected to identify, extend, and create patterns of sounds, physical movement, and concrete objects.

(K.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns to make predictions. The student is expected to:
(A) use patterns to predict what comes next, including cause-and-effect relationships; and
(B) count by ones to 100.

(K.7) **Geometry and spatial reasoning.** The student describes the relative positions of objects. The student is expected to:
(A) describe one object in relation to another using informal language such as over, under, above, and below; and
(B) place an object in a specified position.

(K.8) **Geometry and spatial reasoning.** The student uses attributes to determine how objects are alike and different. The student is expected to:
(A) describe and identify an object by its attributes using informal language;
(B) compare two objects based on their attributes; and
(C) sort a variety of objects including two- and three-dimensional geometric figures according to their attributes and describe how the objects are sorted [those groups are formed].

(K.9) **Geometry and spatial reasoning.** The student recognizes attributes [characteristics] of two- and three-dimensional geometric figures [shapes and solids]. The student is expected to:
(A) describe and compare the attributes of real-life objects such as balls, boxes, cans, and cones or models of three-dimensional geometric figures [solids];
(B) recognize shapes in real-life three-dimensional geometric figures [objects] or models of three-dimensional geometric figures [solids]; and
(C) describe, identify, and compare circles, triangles, [and] rectangles , and [including] squares (a special type of rectangle).

(K.10) **Measurement.** The student directly compares the attributes of [such as] length, area, weight/mass, [weight, or] capacity , and/or relative temperature [to compare and order objects]. The student uses comparative language to solve problems and answer questions. The student is expected to:
(A) compare and order two or three concrete objects according to length (longer/shorter than, or the same); [shorter or longer], capacity (holds more or holds less), or weight (lighter or heavier); and
(B) compare the areas of two flat surfaces of two-dimensional figures (covers more, covers less, or covers the same); [B] find concrete objects that are about the same as, less than, or greater than a given object according to length, capacity, or weight.]
(K.11) **Measurement.** The student uses time [and temperature] to describe, compare, and order events [and situations] and objects.

The student is expected to:

(A) compare situations or objects according to temperature such as hotter or colder;

(B) compare events according to duration such as more time than or less time than;

(C) sequence events (up to three); and

(D) read a calendar using days, weeks, and months.

(K.12) **Probability and statistics.** The student constructs and uses graphs of real objects or pictures to answer questions.

The student is expected to:

(A) construct graphs using real objects or pictures in order to answer questions; and

(B) use information from a graph of real objects or pictures in order to answer questions.

(K.13) **Underlying processes and mathematical tools.** The student applies Kindergarten mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

(A) identify mathematics in everyday situations;

(B) solve problems [use a problem-solving model] with guidance [that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness];

(C) select or develop an appropriate problem-solving strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and

(D) use tools such as real objects, manipulatives, and technology to solve problems.

(K.14) **Underlying processes and mathematical tools.** The student communicates about Kindergarten mathematics using informal language.

The student is expected to:

(A) communicate mathematical ideas [explain and record observations] using objects, words, pictures, numbers, and technology; and

(B) relate everyday language to mathematical language and symbols.

(K.15) **Underlying processes and mathematical tools.** The student uses logical reasoning [to make sense of his or her world]

The student is expected to justify [reason and support] his or her thinking using objects, words, pictures, numbers, and technology.
§111.13. Mathematics, Grade 1.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 1 are building number sense through number relationships, adding and subtracting whole numbers, organizing and analyzing data, and working with two- and three-dimensional geometric figures.

(2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use patterns to describe objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal language and observation of geometric properties to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.

(3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.

(1.1) **Number, operation, and quantitative reasoning.** The student uses whole numbers to describe and compare quantities.

The student is expected to:

(A) compare and order whole numbers up to 99 (less than, greater than, or equal to) using sets of concrete objects and pictorial models;

(B) create sets of tens and ones using concrete objects to describe, compare, and order whole numbers;

(C) identify individual coins by name and value and describe relationships among them; and

(D) read and write numbers to 99 to describe sets of concrete objects.

(1.2) **Number, operation, and quantitative reasoning.** The student uses pairs of whole numbers to describe fractional parts of whole objects or sets of objects.

The student is expected to:

(A) separate a whole [by separating it] into two, three, or four equal parts and use appropriate language to describe the parts such as three out of four equal parts; and

(B) use appropriate language to describe part of a set such as three out of the eight crayons are red.
(1.3) **Number, operation, and quantitative reasoning.** The student recognizes and solves problems in addition and subtraction situations.

The student is expected to:

(A) model and create addition and subtraction problem situations with concrete objects and write corresponding number sentences; and

(B) use concrete and pictorial models to learn and apply basic addition and subtraction facts (up to 9 + 9 = 18 and 18 – 9 = 9) using concrete models.

(1.4) **Patterns, relationships, and algebraic thinking.** The student uses repeating patterns and additive patterns to make predictions.

The student is expected to identify, describe, and extend concrete and pictorial patterns in order to make predictions and solve problems:

(A) identify, describe, and extend concrete and pictorial patterns in order to make predictions and solve problems; and

(B) use patterns to skip count by twos, fives, and tens.

(1.5) **Patterns, relationships, and algebraic thinking.** The student recognizes patterns in numbers and operations.

The student is expected to:

(A) use patterns to skip count by twos, fives, and tens;

(B) find patterns in numbers, including odd and even;

(C) compare and order whole numbers using place value;

(D) use patterns to develop strategies to solve basic addition and basic subtraction problems; and

(E) identify patterns in related addition and subtraction sentences (fact families for sums to 18) such as 2 + 3 = 5, 3 + 2 = 5, 5 – 2 = 3, and 5 – 3 = 2.

(1.6) **Geometry and spatial reasoning.** The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both.

The student is expected to:

(A) describe and identify objects in order to sort them according to a given attribute using informal language;

(B) describe and identify two-dimensional geometric figures, including circles, triangles, rectangles, and squares (a special type of rectangle); and describe the shape of balls, boxes, cans, and cones;

(C) describe and identify two- and three-dimensional geometric figures in order to sort them according to a given attribute using informal and formal language; and

(D) use concrete models to combine two-dimensional geometric figures to make new geometric figures.

(1.7) **Measurement.** The student directly compares the attributes of length, area, weight/mass, capacity, and temperature. The student uses comparative language to solve problems and answer questions. The student selects and uses nonstandard units to describe length, weight, and capacity.

The student is expected to:

(A) estimate and measure length, capacity, and weight of objects using nonstandard units such as paper clips or sides of color tiles;

(B) compare and order two or more concrete objects according to length (from longest to shortest);

(C) describe the relationship between the size of the unit and the number of units needed to measure the length of an object in a measurement.
(D) compare and order the area of two or more two-dimensional surfaces (from covers the most to covers the least);
(E) compare and order two or more containers according to capacity (from holds the most to holds the least);
(F) compare and order two or more objects according to weight/mass (from heaviest to lightest); and
(G) compare and order two or more objects according to relative temperature (from hottest to coldest).

(1.8) **Measurement.** The student understands that time and temperature can be measured. The student uses time to describe and compare situations.

The student is expected to:
(A) recognize temperatures such as a hot day or a cold day;
(B) describe time on a clock using hours and half hours; and
(C) order three or more events according to duration; and by how much time they take.

(B) read time to the hour and half-hour using analog and digital clocks.

(1.9) **Probability and statistics.** The student displays data in an organized form.

The student is expected to:
(A) collect and sort data; and
(B) use organized data to construct real-object graphs, picture graphs, and bar-type graphs.

(1.10) **Probability and statistics.** The student uses information from organized data.

The student is expected to:
(A) draw conclusions and answer questions using information organized in real-object graphs, picture graphs, and bar-type graphs; and
(B) identify events as certain or impossible such as drawing a red crayon from a bag of green crayons.

(1.11) **Underlying processes and mathematical tools.** The student applies Grade 1 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:
(A) identify mathematics in everyday situations;
(B) solve problems [use a problem-solving model] with guidance [as needed] that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving plan or strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and
(D) use tools such as real objects, manipulatives, and technology to solve problems.

(1.12) **Underlying processes and mathematical tools.** The student communicates about Grade 1 mathematics using informal language.

The student is expected to:
(A) explain and record observations using objects, words, pictures, numbers, and technology; and
(B) relate informal language to mathematical language and symbols.

(1.13) **Underlying processes and mathematical tools.** The student uses logical reasoning [to make sense of his or her world].

The student is expected to justify [reason and support] his or her thinking using objects, words, pictures, numbers, and technology.
(a) Introduction.
(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 2 are developing an understanding of the base-ten place value system, comparing and ordering whole numbers, applying addition and subtraction, and using measurement processes.
(2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use patterns to describe objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal language and observation of geometric properties to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.
(3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.
(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.
(2.1) **Number, operation, and quantitative reasoning.** The student understands how place value is used to represent whole numbers.

The student is expected to: use concrete models to represent, compare, and order whole numbers (through 999); read the numbers; and record the comparisons using numbers and symbols (>, <, =).

(A) use concrete models of hundreds, tens, and ones to represent a given whole number (up to 999) in various ways;
(B) use place value to read, write, and describe the value of whole numbers to 999; and
(C) use place value to compare and order whole numbers to 999 and record the comparisons using numbers and symbols (<, =, >).

(2.2) **Number, operation, and quantitative reasoning.** The student describes how fractions are used to name parts of whole objects or sets of objects.

The student is expected to:
(A) use concrete models to represent and name fractional parts of a whole object (with denominators of 12 or less); (not to exceed twelfths) when given a concrete representation; and
(B) use concrete models to represent and name fractional parts of a set of objects (with denominators of 12 or less); and (not to exceed twelfths) when given a concrete representation;
(C) use concrete models to determine if a fractional part of a whole is closer to 0, ½, or 1.
(2.3) **Number, operation, and quantitative reasoning.** The student adds and subtracts whole numbers to solve problems.

The student is expected to:

(A) recall and apply basic addition and subtraction facts (sums to 18);

(B) model addition and subtraction of two-digit numbers with objects, pictures, words, and numbers;

(C) select addition or subtraction to solve problems using two-digit numbers, whether or not regrouping is necessary;

(D) determine the value of a collection of coins up to one dollar; and

(E) describe how the cent symbol, dollar symbol, and the decimal point are used to name the value of a collection of coins.

(2.4) **Number, operation, and quantitative reasoning.** The student models multiplication and division.

The student is expected to:

(A) model, create, and describe multiplication situations in which equivalent sets of concrete objects are joined; and

(B) model, create, and describe division situations in which a set of concrete objects is separated into equivalent sets.

(2.5) **Patterns, relationships, and algebraic thinking.** The student uses patterns in numbers and operations.

The student is expected to:

(A) find patterns in numbers such as in a 100s chart;

(B) use patterns in place value to compare and order whole numbers through 999; and

(C) use patterns and relationships to develop strategies to remember basic addition and subtraction facts.

Determine patterns in related addition and subtraction number sentences (including fact families) such as $8 + 9 = 17, 9 + 8 = 17, 17 - 8 = 9, and 17 - 9 = 8$.

(D) solve subtraction problems related to addition facts (fact families) such as $8 + 9 = 17, 9 + 8 = 17, 17 - 8 = 9, and 17 - 9 = 8$.

(2.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns to describe relationships and make predictions.

The student is expected to:

(A) generate a list of paired numbers based on a real-life situation such as number of tricycles related to number of wheels;

(B) identify patterns in a list of related number pairs based on a real-life situation and extend the list; and

(C) identify, describe, and extend repeating and additive patterns to make predictions and solve problems.

(2.7) **Geometry and spatial reasoning.** The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both.

The student is expected to:

(A) describe attributes (the number of vertices, faces, edges, sides) of two- and three-dimensional geometric figures such as circles, polygons, spheres, cones, cylinders, prisms, and pyramids, etc.

(B) use attributes to describe how two two-dimensional figures or two three-dimensional geometric figures are alike or different; and

(C) cut two-dimensional geometric figures apart and identify the new geometric figures formed.
(2.8) **Geometry and spatial reasoning.** The student recognizes that numbers can be represented by points on a line and can be used to represent a set of numbers and its properties.

The student is expected to use whole numbers to locate and name points on a number line.

(2.9) **Measurement.** The student directly compares the attributes of length, area, weight/mass, and capacity, and uses comparative language to solve problems and answer questions. The student selects and uses nonstandard units to describe length, area, capacity, and weight/mass. The student recognizes and uses models that approximate standard units (from both SI, also known as metric, and customary systems) of length, weight/mass, capacity, and time.

The student is expected to:

(A) identify concrete models that approximate standard units of length and use them to measure length, capacity, and weight;

(B) select a non-standard unit of measure such as square tiles to determine the area of a two-dimensional surface;

(C) measure length, capacity, and weight using concrete models that approximate standard units; and

(D) select a non-standard unit of measure such as beans or marbles to determine the weight/mass of a given object.

(2.10) **Measurement.** The student uses standard tools to estimate and measure time and temperature (in degrees Fahrenheit).

The student is expected to:

(A) read and write times shown on analog and digital clocks using five-minute increments; and

(C) describe activities that take approximately one second, one minute, and one hour.

(2.11) **Probability and statistics.** The student organizes data to make it useful for interpreting information.

The student is expected to:

(A) construct picture graphs and bar-type graphs;

(B) draw conclusions and answer questions based on picture graphs and bar-type graphs; and

(C) use data to describe events as more likely or less likely such as drawing a certain color crayon from a bag of seven red crayons and three green crayons.

(2.12) **Underlying processes and mathematical tools.** The student applies Grade 2 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

(A) identify the mathematics in everyday situations;

(B) solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving plan or strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and

(D) use tools such as real objects, manipulatives, and technology to solve problems.

(2.13) **Underlying processes and mathematical tools.** The student communicates about Grade 2 mathematics using informal language and tools.

The student is expected to:

(A) explain and record observations using objects, words, pictures, numbers, and technology; and

(B) relate informal language to mathematical language and tools.
language. symbols.

(2.14) Underlying processes and mathematical tools. The student uses logical reasoning to make sense of his or her thinking using objects, words, pictures, numbers, and technology.

§111.15. Mathematics, Grade 3.
(a) Introduction.
(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 3 are multiplying and dividing whole numbers, connecting fraction symbols to fractional quantities, and standardizing language and procedures in geometry and measurement.

(2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.

(3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.

(b) Knowledge and skills.
(3.1) Number, operation, and quantitative reasoning. The student uses place value to communicate about increasingly large whole numbers in verbal and written form, including money. The student is expected to:
(A) use place value to read, write (in symbols and words), and describe the value of whole numbers through 999,999;
(B) use place value to compare and order whole numbers through 9,999; and
(C) determine the value of a collection of coins and bills.

(3.2) Number, operation, and quantitative reasoning. The student uses fraction names and symbols (with denominators of 12 or less) to describe fractional parts of whole objects or sets of objects. The student is expected to:
(A) construct concrete models of fractions;
(B) compare fractional parts of whole objects or sets of objects in a problem situation using concrete models;
(C) use fraction names and symbols to describe fractional parts of whole objects or sets of objects [with denominators of 12 or less]; and
(D) construct concrete models of equivalent fractions for fractional parts of whole objects.
(3.3) **Number, operation, and quantitative reasoning.** The student adds and subtracts to solve meaningful problems involving whole numbers.

The student is expected to:

(A) model addition and subtraction using pictures, words, and numbers; and

(B) select addition or subtraction and use the operation to solve problems involving whole numbers through 999.

(3.4) **Number, operation, and quantitative reasoning.** The student recognizes and solves problems in multiplication and division situations.

The student is expected to:

(A) learn and apply multiplication facts through 12 by 12 [the tens] using concrete models and objects;

(B) solve and record multiplication problems (up to two digits times one digit) [(one-digit multiplier)]; and

(C) use models to solve division problems and use number sentences to record the solutions.

(3.5) **Number, operation, and quantitative reasoning.** The student estimates to determine reasonable results.

The student is expected to:

(A) round whole numbers [two-digit numbers] to the nearest ten or hundred to approximate reasonable results in problem situations [and three-digit numbers to the nearest hundred]; and

(B) use strategies including rounding and compatible numbers to estimate solutions to addition and subtraction problems.

[(B) estimate sums and differences beyond basic facts.]

(3.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns to solve problems.

The student is expected to:

(A) identify and extend whole-number and geometric patterns to make predictions and solve problems;

(B) identify patterns in multiplication facts using concrete objects, pictorial models, or technology; and

(C) identify patterns in related multiplication and division sentences (fact families) such as 2 x 3 = 6, 3 x 2 = 6, 6 ÷ 2 = 3, 6 ÷ 3 = 2.

(3.7) **Patterns, relationships, and algebraic thinking.** The student uses lists, tables, and charts to express patterns and relationships.

The student is expected to:

(A) generate a table of paired numbers based on a real-life situation such as insects and legs; and

(B) identify and describe patterns in a table of related number pairs based on a meaningful problem [real-life situation] and extend the table.

(3.8) **Geometry and spatial reasoning.** The student uses formal geometric vocabulary.

The student is expected to identify, classify, and [name] describe two- and three-dimensional geometric figures by their attributes. The student compares two-dimensional figures, three-dimensional figures, or both by their attributes and compare shapes and solids using formal geometry [geometric] vocabulary.

(3.9) **Geometry and spatial reasoning.** The student recognizes congruence and symmetry.

The student is expected to:

(A) identify congruent two-dimensional figures [shapes];

(B) create two-dimensional figures [shapes] with lines of symmetry using concrete models and technology; and

(C) identify lines of symmetry in two-dimensional geometric figures [shapes].

(3.10) **Geometry and spatial reasoning.** The student recognizes that

The student is expected to locate and name points on a number line using whole numbers and fractions, including
numbers can be represented by points on a line can be used to represent numbers and fractions and their properties and relationships.

(3.11) Measurement. The student directly compares the attributes of length, area, weight/mass, and capacity, and uses comparative language to solve problems and answer questions. The student selects and uses standard units to describe length, area, capacity/volume, and weight/mass.

The student is expected to:
(A) use linear measurement tools to estimate and measure lengths using standard units [such as inch, foot, yard, centimeter, decimeter, and meter];
(B) use standard units [linear measure] to find the perimeter of a shape; [and]
(C) use concrete and pictorial models of square units to determine the area of two-dimensional surfaces; [shapes.]
(D) identify concrete models that approximate standard units of weight/mass and use them to measure weight/mass; [E] identify concrete models that approximate standard units for capacity and use them to measure capacity; and
(F) use concrete models that approximate cubic units to determine the volume of a given container or other three-dimensional geometric figure.

(3.12) Measurement. The student reads and writes time and measures [time and] temperature in degrees Fahrenheit to solve problems.

The student is expected to:
(A) use a thermometer to measure temperature; and
(B) tell and write time shown on analog [traditional] and digital clocks, [and]
(B) use a thermometer to measure temperature.]

(3.13) Measurement. The student applies measurement concepts.]

The student is expected to measure to solve problems involving length, area, temperature, and time.

The student is expected to:
(A) collect, organize, record, and display data in pictographs and bar graphs where each picture or cell might represent more than one piece of data;
(B) interpret information from pictographs and bar graphs; and
(C) use data to describe events as more likely than, less likely than, or equally likely as.

(3.14) Underlying processes and mathematical tools. The student applies Grade 3 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:
(A) identify the mathematics in everyday situations;
(B) solve problems [use a problem-solving model] that incorporate [incorporates] understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
(D) use tools such as real objects, manipulatives, and technology to solve problems.

(3.15) Underlying processes and mathematical tools. The student is expected to:
(A) explain and record observations using objects, words,
student communicates about Grade 3 mathematics using informal language.

(B) relate informal language to mathematical language and symbols.

(3.16) Underlying processes and mathematical tools. The student uses logical reasoning to make sense of his or her world.

The student is expected to:

(A) make generalizations from patterns or sets of examples and nonexamples; and

(B) justify why an answer is reasonable and explain the solution process.

§111.16. Mathematics, Grade 4.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 4 are comparing and ordering fractions and decimals, applying multiplication and division, and developing ideas related to congruence and symmetry.

(2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.

(3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 3-5, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.

(4.1) Number, operation, and quantitative reasoning. The student uses place value to represent whole numbers and decimals.

The student is expected to:

(A) use place value to read, write, compare, and order whole numbers through 999,999,999 [the millions place]; and

(B) use place value to read, write, compare, and order decimals involving tenths and hundredths, including money, using concrete objects and pictorial models.

(4.2) Number, operation, and quantitative reasoning. The student describes and compares fractional parts of whole objects or sets of objects.

The student is expected to:

(A) use concrete objects and pictorial models to generate equivalent fractions [using concrete and pictorial models];

(B) model fraction quantities greater than one using concrete objects and pictorial models [materials and pictures];

(C) compare and order fractions using concrete objects and
pictorial models; and
(D) relate decimals to fractions that name tenths and hundredths using concrete objects and pictorial models.

(4.3) **Number, operation, and quantitative reasoning.** The student adds and subtracts to solve meaningful problems involving whole numbers and decimals.

The student is expected to:
(A) use addition and subtraction to solve problems involving whole numbers; and
(B) add and subtract decimals to the hundredths place using concrete objects and pictorial models.

(4.4) **Number, operation, and quantitative reasoning.** The student multiplies and divides to solve meaningful problems involving whole numbers.

The student is expected to:
(A) model factors and products using arrays and area models;
(B) represent multiplication and division situations in picture, word, and number form;
(C) recall and apply multiplication facts through 12 x 12;
(D) use multiplication to solve problems (no more than two digits times two digits without technology) involving two-digit numbers; and
(E) use division to solve problems (no more than one-digit divisors and three-digit dividends without technology) involving one-digit divisors.

(4.5) **Number, operation, and quantitative reasoning.** The student estimates to determine reasonable results.

The student is expected to:
(A) round whole numbers to the nearest ten, hundred, or thousand to approximate reasonable results in problem situations; and
(B) use strategies including rounding and compatible numbers to estimate solutions to multiplication and division problems.

(B) estimate a product or quotient beyond basic facts.

(4.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns in multiplication and division.

The student is expected to:
(A) use patterns and relationships to develop strategies to remember basic multiplication and division facts (such as the patterns in related multiplication and division number sentences (fact families) such as 9 x 9 = 81 and 81 + 9 = 9); and
(B) solve division problems related to multiplication facts (fact families) such as 9 x 9 = 81 and 81 ÷ 9 = 9; and
(B) use patterns to multiply by 10 and 100.

(4.7) **Patterns, relationships, and algebraic thinking.** The student uses organizational structures to analyze and describe patterns and relationships.

The student is expected to describe the relationship between two sets of related data such as ordered pairs in a table.

(4.8) **Geometry and spatial reasoning.** The student identifies and describes attributes of geometric figures (lines, shapes, and solids) using formal geometric language.

The student is expected to:
(A) identify and describe right, acute, and obtuse angles;
(B) identify and describe models of parallel and intersecting (including perpendicular) lines using concrete objects and pictorial models; and
(C) use essential attributes to define two- and three-dimensional geometric figures.
(C) describe shapes and solids in terms of vertices, edges,
(4.9) **Geometry and spatial reasoning.**
The student connects transformations to congruence and symmetry.

The student is expected to:
(A) demonstrate translations, reflections, and rotations using concrete models;
(B) use translations, reflections, and rotations to verify that two shapes are congruent; and
(C) use reflections to verify that a shape has symmetry.

(4.10) **Geometry and spatial reasoning.**
The student recognizes the connection between numbers and their properties and points on a number line.

The student is expected to locate and name points on a number line using whole numbers, fractions such as halves and fourths, and decimals such as tenths.

(4.11) **Measurement.**
The student applies measurement concepts.

The student is expected to:
(A) estimate and use measurement tools to determine length (including perimeter), area, capacity and weight/mass using standard units SI (metric) and customary;
(B) perform simple conversions between different units of length, between different units of capacity, and between different units of weight within the customary measurement system;
(C) use concrete models of standard cubic units to measure volume;
(D) estimate volume in cubic units; and
(E) explain the difference between weight and mass.

(4.12) **Measurement.**
The student applies measurement concepts.

The student measures time and temperature (in degrees Fahrenheit and Celsius).

The student is expected to:
(A) use a thermometer to measure temperature and changes in temperature; and
(B) use tools such as a clock with gears or a stopwatch to solve problems involving elapsed time.

(4.13) **Probability and statistics.**
The student solves problems by collecting, organizing, displaying, and interpreting sets of data.

The student is expected to:
(A) use concrete objects or pictures to make generalizations about determining all possible combinations of a given set of data or of objects in a problem situation; and
[(A)] list all possible outcomes of a probability experiment such as tossing a coin;
[(B)] use a pair of numbers to compare favorable outcomes to all possible outcomes such as four heads out of six tosses of a coin; and
(B) [(C)] interpret bar graphs.
(4.14) **Underlying processes and mathematical tools.** The student applies Grade 4 mathematics to solve problems connected to everyday experiences and activities in and outside of school. The student is expected to:

(A) identify the mathematics in everyday situations;

(B) solve problems [use a problem-solving model] that incorporate [incorporates] understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

(D) use tools such as real objects, manipulatives, and technology to solve problems.

(4.15) **Underlying processes and mathematical tools.** The student communicates about Grade 4 mathematics using informal language. The student is expected to:

(A) explain and record observations using objects, words, pictures, numbers, and technology; and

(B) relate informal language to mathematical language and symbols.

(4.16) **Underlying processes and mathematical tools.** The student uses logical reasoning [to make sense of his or her world]. The student is expected to:

(A) make generalizations from patterns or sets of examples and non-examples; and

(B) justify why an answer is reasonable and explain the solution process.

§111.17. Mathematics, Grade 5.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 5 are comparing and contrasting lengths, areas [area], and volumes [volume] of two- or three-dimensional geometric figures [geometric shapes and solids]; representing and interpreting data in graphs, charts, and tables; and applying whole number operations in a variety of contexts.

(2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures [shapes and solids]; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.

(3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 3-5, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.
(b) Knowledge and skills.

(5.1) **Number, operation, and quantitative reasoning.** The student uses place value to represent whole numbers and decimals.

The student is expected to:

(A) use place value to read, write, compare, and order whole numbers through the \(999,999,999\) [billions place]; and

(B) use place value to read, write, compare, and order decimals through the thousandths place.

(5.2) **Number, operation, and quantitative reasoning.** The student uses fractions in problem-solving situations.

The student is expected to:

(A) generate a fraction equivalent to a given fraction such as \(\frac{1}{2}\) and \(\frac{3}{6}\) or \(\frac{4}{12}\) and \(\frac{1}{3}\); 
(B) generate equivalent fractions;

(C) compare two fractional quantities in problem-solving situations using a variety of methods, including common denominators; and

(D) use models to relate decimals to fractions that name tenths, hundredths, and thousandths.

(5.3) **Number, operation, and quantitative reasoning.** The student adds, subtracts, multiplies, and divides to solve meaningful problems.

The student is expected to:

(A) use addition and subtraction to solve problems involving whole numbers and decimals;

(B) use multiplication to solve problems involving whole numbers (no more than three digits times two digits without technology);

(C) use division to solve problems involving whole numbers (no more than two-digit divisors and three-digit dividends without technology), including interpreting the remainder within a given context;

(D) identify prime factors of a whole number and common factors of a set of whole numbers; and

(E) model situations using addition and/or subtraction involving fractions with like denominators using concrete objects, pictures, words, and numbers in problem-solving situations.

(5.4) **Number, operation, and quantitative reasoning.** The student estimates to determine reasonable results.

The student is expected to use strategies, including rounding and compatible numbers to estimate solutions to addition, subtraction, multiplication, and division problems.

(A) round whole numbers and decimals through tenths to approximate reasonable results in problem situations; and

(B) estimate to solve problems where exact answers are not required.

(5.5) **Patterns, relationships, and algebraic thinking.** The student makes generalizations based on observed patterns and relationships.

The student is expected to:

(A) use concrete objects or pictures to make generalizations about determining all possible combinations;

(B) describe the relationship between sets of data in graphic organizers such as lists, tables, charts, and diagrams to find patterns and make generalizations such as a procedure for determining equivalent fractions; and
(5.6) **Patterns, relationships, and algebraic thinking.** The student describes relationships mathematically.

The student is expected to select from and use diagrams and equations such as \( y = 5 + 3 \) [number sentences] to represent meaningful problem [real-life] situations.

(B) [(C)] Identify prime and composite numbers using concrete objects, pictorial models, and patterns in factor pairs.

(5.7) **Geometry and spatial reasoning.** The student generates geometric definitions using critical attributes.

The student is expected to identify essential attributes including parallel, perpendicular, and congruent parts of two- and three-dimensional geometric figures. 

[(A)] identify critical attributes including parallel, perpendicular, and congruent parts of geometric shapes and solids; and

[(B)] use critical attributes to define geometric shapes or solids.

(5.8) **Geometry and spatial reasoning.** The student models transformations.

The student is expected to:

(A) sketch the results of translations, rotations, and reflections on a Quadrant I coordinate grid; and

(B) identify [describe] the transformation that generates one figure from the other when given two congruent figures on a Quadrant I coordinate grid.

(5.9) **Geometry and spatial reasoning.** The student recognizes the connection between ordered pairs of numbers and locations of points on a plane.

The student is expected to locate and name points on a coordinate grid using ordered pairs of whole numbers.

(5.10) **Measurement.** The student applies measurement concepts involving length (including perimeter), area, capacity/volume, and weight/mass to solve problems [selects and uses appropriate units and procedures to measure volume].

The student is expected to:

(A) perform simple conversions within the same measurement system (SI (metric) or customary):

[(A)] measure volume using concrete models of cubic units; and

(B) connect models for perimeter, area, and volume with their respective formulas; and

[(B)] estimate volume in cubic units.

(C) select and use appropriate units and formulas to measure length, perimeter, area, and volume.

(5.11) **Measurement.** The student applies measurement concepts. The student measures time and temperature (in degrees Fahrenheit and Celsius).

The student is expected to:

(A) solve problems involving changes in temperature; and

[(A)] measure to solve problems involving length (including perimeter), weight, capacity, time, temperature, and area; and

(B) solve problems involving elapsed time.

[(B)] describe numerical relationships between units of measure within the same measurement system such as an inch is one-twelfth of a foot.

(5.12) **Probability and statistics.** The student describes and predicts the results of a probability experiment.

The student is expected to:

(A) use fractions to describe the results of an experiment; and

[(B)] use experimental results to make predictions; and [(C)] list all possible outcomes of a probability experiment such
(5.13) **Probability and statistics.** The student solves problems by collecting, organizing, displaying, and interpreting sets of data. The student is expected to:
(A) use tables of related number pairs to make line graphs;
(B) describe characteristics of data presented in tables and graphs including median, mode, and range; and
(C) graph a given set of data using an appropriate graphical representation such as a picture or line graph.

(5.14) **Underlying processes and mathematical tools.** The student applies Grade 5 mathematics to solve problems connected to everyday experiences and activities in and outside of school. The student is expected to:
(A) identify the mathematics in everyday situations;
(B) solve problems that incorporate understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
(D) use tools such as real objects, manipulatives, and technology to solve problems.

(5.15) **Underlying processes and mathematical tools.** The student communicates about Grade 5 mathematics using informal language. The student is expected to:
(A) explain and record observations using objects, words, pictures, numbers, and technology; and
(B) relate informal language to mathematical language and symbols.

(5.16) **Underlying processes and mathematical tools.** The student uses logical reasoning to make sense of his or her world. The student is expected to:
(A) make generalizations from patterns or sets of examples and non-examples; and
(B) justify why an answer is reasonable and explain the solution process.

§111.22. Mathematics, Grade 6.
(a) Introduction.
(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 6 are using
ratios to describe direct proportional relationships involving number, geometry, measurement, probability and adding and subtracting decimals and fractions.
(2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.
(3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in
mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology ([at least four-function calculators for whole numbers, decimals, and fractions]) and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.

(b) Knowledge and skills.

(6.1) **Number, operation, and quantitative reasoning.** The student represents and uses rational numbers in a variety of equivalent forms.

The student is expected to:

(A) compare and order non-negative rational numbers;

(B) generate equivalent forms of rational numbers including whole numbers, fractions, and decimals;

(C) use integers to represent real-life situations;

(D) write prime factorizations using exponents; [and]

(E) identify factors of a positive integer, [and multiples including] common factors, and the greatest common factor of a set of positive integers; and [common multiples.]

(F) identify multiples of a positive integer and common multiples and the least common multiple of a set of positive integers.

(6.2) **Number, operation, and quantitative reasoning.** The student adds, subtracts, multiplies, and divides to solve problems and justify solutions.

The student is expected to:

(A) model addition and subtraction situations involving fractions with objects, pictures, words, and numbers;

(B) use addition and subtraction to solve problems involving fractions and decimals;

(C) use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates; [and]

(D) estimate and round to approximate reasonable results and to solve problems where exact answers are not required; and [;

(E) use order of operations to simplify whole number expressions (without exponents) in problem solving situations.

(6.3) **Patterns, relationships, and algebraic thinking.** The student solves problems involving direct proportional relationships.

The student is expected to:

(A) use ratios to describe proportional situations;

(B) represent ratios and percents with concrete models, fractions, and decimals; and

(C) use ratios to make predictions in proportional situations.

(6.4) **Patterns, relationships, and algebraic thinking.** The student uses letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes.

The student is expected to:

(A) use tables and symbols to represent and describe proportional and other relationships such as those involving conversions, arithmetic sequences (with a constant rate of change), perimeter and [;] area [;etc.;] and

(B) use tables of data to generate formulas representing [to represent] relationships involving perimeter, area, volume of a rectangular prism, etc. [from a table of data.]

(6.5) **Patterns, relationships, and algebraic thinking.** The student uses letters to represent an unknown in an equation.

The student is expected to formulate equations [an equation] from [a] problem situations described by linear relationships [situation].
(6.6) **Geometry and spatial reasoning.** The student uses geometric vocabulary to describe angles, polygons, and circles.

The student is expected to:

(A) use angle measurements to classify angles as acute, obtuse, or right;
(B) identify relationships involving angles in triangles and quadrilaterals; and
(C) describe the relationship between radius, diameter, and circumference of a circle.

(6.7) **Geometry and spatial reasoning.** The student uses coordinate geometry to identify location in two dimensions.

The student is expected to locate and name points on a coordinate plane using ordered pairs of non-negative rational numbers.

(6.8) **Measurement.** The student solves application problems involving estimation and measurement of length, area, time, temperature, volume [capacity], weight, and angles.

The student is expected to:

(A) estimate measurements (including circumference) and evaluate reasonableness of results;
(B) select and use appropriate units, tools, or formulas to measure and to solve problems involving length (including perimeter and circumference), area, time, temperature, volume [capacity], and weight;
(C) measure angles; and
(D) convert measures within the same measurement system (customary and metric) based on relationships between units.

(6.9) **Probability and statistics.** The student uses experimental and theoretical probability to make predictions.

The student is expected to:

(A) construct sample spaces using lists and tree diagrams and combinations; and
(B) find the probabilities of a simple event and its complement and describe the relationship between the two.

(6.10) **Probability and statistics.** The student uses statistical representations to analyze data.

The student is expected to:

(A) select and use an appropriate representation for presenting and displaying different graphical representations of the same data including line plot, line graph, bar graph, and stem and leaf plot;
(B) identify mean (using concrete objects and pictorial models), median, mode, and range of a set of data; and
(C) sketch circle graphs to display data; and
(D) solve problems by collecting, organizing, displaying, and interpreting data.

(6.11) **Underlying processes and mathematical tools.** The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school.

The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;
(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic
guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.

(6.12) **Underlying processes and mathematical tools.** The student communicates about Grade 6 mathematics through informal and mathematical language, representations, and models.

The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and

(B) evaluate the effectiveness of different representations to communicate ideas.

(6.13) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions.

The student is expected to:

(A) make conjectures from patterns or sets of examples and non-examples; and

(B) validate his/her conclusions using mathematical properties and relationships.

§111.23. Mathematics, Grade 7.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 7 are using direct proportional relationships in number, geometry, measurement, and probability; applying addition, subtraction, multiplication, and division of decimals, fractions, and integers; and using statistical measures to describe data.

(2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures [objects] or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.

(3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology [at least four function calculators for whole numbers, decimals, and fractions] and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.

(b) Knowledge and skills.

(7.1) **Number, operation, and quantitative reasoning.** The student represents and uses numbers in a variety of equivalent forms.

The student is expected to:

(A) compare and order integers and positive rational numbers;

(B) convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator; and

(C) represent squares and square roots using geometric
(7.2) **Number, operation, and quantitative reasoning.** The student adds, subtracts, multiplies, or divides to solve problems and justify solutions.

The student is expected to:
(A) represent multiplication and division situations involving fractions and decimals with **concrete** models, including concrete objects, pictures, words, and numbers;
(B) use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals;
(C) use models, such as concrete objects, pictorial models, and number lines, to add, subtract, multiply, and divide integers and connect the actions to algorithms;
(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio;
(E) simplify numerical expressions involving order of operations and exponents;
(F) select and use appropriate operations to solve problems and justify the selections; and
(G) determine the reasonableness of a solution to a problem

(7.3) **Patterns, relationships, and algebraic thinking.** The student solves problems involving direct proportional relationships.

The student is expected to:
(A) estimate and find solutions to application problems involving percent; and
(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.

(7.4) **Patterns, relationships, and algebraic thinking.** The student represents a relationship in numerical, geometric, verbal, and symbolic form.

The student is expected to:
(A) generate formulas involving unit conversions, perimeter, area, circumference, volume, and scaling;
(B) graph data to demonstrate relationships in familiar concepts such as conversions, perimeter, area, circumference, volume, and scaling; and
(C) use words and symbols to describe the relationship between the terms in an arithmetic sequence (with a constant rate of change) and their positions in the sequence

(7.5) **Patterns, relationships, and algebraic thinking.** The student uses equations to solve problems.

The student is expected to:
(A) use concrete and pictorial models to solve equations and use symbols to record the actions; and
(B) formulate **a possible** problem situations **situation** when given a simple equation and formulate an equation when given a problem situation

(7.6) **Geometry and spatial reasoning.** The student compares and classifies two- and three-dimensional figures [shapes and solids] using geometric vocabulary and properties.

The student is expected to:
(A) use angle measurements to classify pairs of angles as complementary or supplementary;
(B) use properties to classify shapes including triangles and quadrilaterals, pentagons, and circles;
(C) use properties to classify three-dimensional figures [solids], including pyramids, cones, prisms, and cylinders; and
(D) use critical attributes to define similarity
(7.7) **Geometry and spatial reasoning.** The student uses coordinate geometry to describe location on a plane.

The student is expected to:
(A) locate and name points on a coordinate plane using ordered pairs of integers; and
(B) graph reflections across the horizontal or vertical axis and graph translations on a coordinate plane.

(7.8) **Geometry and spatial reasoning.** The student uses geometry to model and describe the physical world.

The student is expected to:
(A) sketch three-dimensional figures (a solid) when given the top, side, and front views;
(B) make a net (two-dimensional model) of the surface area of a three-dimensional figure (solid); and
(C) use geometric concepts and properties to solve problems in fields such as art and architecture.

(7.9) **Measurement.** The student solves application problems involving estimation and measurement.

The student is expected to: estimate measurements and solve application problems involving length (including perimeter and circumference), area, and volume.
(A) estimate measurements and solve application problems involving length (including perimeter and circumference) and area of polygons and other shapes;
(B) connect models for volume of prisms (triangular and rectangular) and cylinders to formulas of prisms (triangular and rectangular) and cylinders; and
(C) estimate measurements and solve application problems involving volume of prisms (rectangular and triangular) and cylinders.

(7.10) **Probability and statistics.** The student recognizes that a physical or mathematical model can be used to describe the experimental and theoretical probability of real-life events.

The student is expected to:
(A) construct sample spaces for simple or composite experiments (compound events (dependent and independent)); and
(B) find the approximate probability of independent events (a compound event through experimentation).

(7.11) **Probability and statistics.** The student understands that the way a set of data is displayed influences its interpretation.

The student is expected to:
(A) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plot, line graph, bar graph, stem and leaf plot, circle graph, and Venn diagrams, and justify the selection; and
(B) make inferences and convincing arguments based on an analysis of given or collected data.

(7.12) **Probability and statistics.** The student uses measures of central tendency and range to describe a set of data.

The student is expected to:
(A) describe a set of data using mean, median, mode, and range; and
(B) choose among mean, median, mode, or range to describe a set of data and justify the choice for a particular situation.

(7.13) **Underlying processes and mathematical tools.** The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and with other disciplines, and with other mathematical topics.

The student is expected to:
(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;
(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out
and activities in and outside of school.

the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems

(7.14) Underlying processes and mathematical tools. The student communicates about Grade 7 mathematics through informal and mathematical language, representations, and models.

The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and

(B) evaluate the effectiveness of different representations to communicate ideas

(7.15) Underlying processes and mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions.

The student is expected to:

(A) make conjectures from patterns or sets of examples and non-examples; and

(B) validate his/her conclusions using mathematical properties and relationships.

§111.24. Mathematics, Grade 8.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 8 are using basic principles of algebra to analyze and represent both proportional and non-proportional linear relationships and using probability to describe data and make predictions.

(2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures [objects] or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.

(3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology [(at least four-function calculators for whole numbers, decimals, and fractions)] and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.
(b) Knowledge and skills.

(8.1) **Number, operation, and quantitative reasoning.** The student understands that different forms of numbers are appropriate for different situations.

The student is expected to:

(A) compare and order rational numbers in various forms including integers, percents, and positive and negative fractions and decimals;

(B) select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships;

(C) approximate (mentally and with calculators) the value of irrational numbers as they arise from problem situations (such as π, √2); and

(D) express numbers in scientific notation, including negative exponents, in appropriate problem situations using a calculator.

(8.2) **Number, operation, and quantitative reasoning.** The student selects and uses appropriate operations to solve problems and justify solutions.

The student is expected to:

(A) select and use appropriate operations to solve problems involving rational numbers and justify the selections;

(B) use appropriate operations to solve problems involving rational numbers in problem situations;

(C) evaluate a solution for reasonableness; and

(D) use multiplication by a constant factor (unit rate) to represent proportional relationships; for example, the arm span of a gibbon is about 1.4 times its height, \( a = 1.4h \).

(8.3) **Patterns, relationships, and algebraic thinking.** The student identifies proportional or non-proportional linear relationships in problem situations and solves problems.

The student is expected to:

(A) compare and contrast proportional and non-proportional linear relationships; and

(B) estimate and find solutions to application problems involving percents and other proportional relationships such as similarity and rates.

(8.4) **Patterns, relationships, and algebraic thinking.** The student makes connections among various representations of a numerical relationship.

The student is expected to generate a different representation of data given another representation of data (such as a table, graph, equation, or verbal description).

(8.5) **Patterns, relationships, and algebraic thinking.** The student uses graphs, tables, and algebraic representations to make predictions and solve problems.

The student is expected to:

(A) predict, estimate, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations; and

(B) find and evaluate an algebraic expression to determine any term in an arithmetic sequence (with a constant rate of change).

(8.6) **Geometry and spatial reasoning.** The student uses transformational geometry to develop spatial sense.

The student is expected to:

(A) generate similar figures using dilations including enlargements and reductions; and

(B) graph dilations, reflections, and translations on a coordinate plane.
(8.7) **Geometry and spatial reasoning.** The student uses geometry to model and describe the physical world. The student is expected to:

(A) draw three-dimensional figures [solids] from different perspectives;

(B) use geometric concepts and properties to solve problems in fields such as art and architecture;

(C) use pictures or models to demonstrate the Pythagorean Theorem; and

(D) locate and name points on a coordinate plane using ordered pairs of rational numbers.

(8.8) **Measurement.** The student uses procedures to determine measures of three-dimensional figures [solids]. The student is expected to:

(A) find lateral and total surface area of prisms, pyramids, and cylinders using concrete models and nets (two-dimensional models);

(B) connect models to formulas for volume of prisms, cylinders, pyramids, spheres, and cones to formulas for volume of these objects; and

(C) estimate measurements and use formulas to solve application problems involving lateral and total surface area and volume.

(8.9) **Measurement.** The student uses indirect measurement to solve problems. The student is expected to:

(A) use the Pythagorean Theorem to solve real-life problems; and

(B) use proportional relationships in similar two-dimensional figures or similar three-dimensional figures [shapes] to find missing measurements.

(8.10) **Measurement.** The student describes how changes in dimensions affect linear, area, and volume measures. The student is expected to:

(A) describe the resulting effects on perimeter and area when dimensions of a shape are changed proportionally; and

(B) describe the resulting effect on volume when dimensions of a solid are changed proportionally.

(8.11) **Probability and statistics.** The student applies concepts of theoretical and experimental probability to make predictions. The student is expected to:

(A) find the probabilities of dependent and independent compound events [dependent and independent];

(B) use theoretical probabilities and experimental results to make predictions and decisions; and

(C) select and use different models to simulate an event.

(8.12) **Probability and statistics.** The student uses statistical procedures to describe data. The student is expected to:

(A) select the appropriate measure of central tendency or range to describe a set of data and justify the choice for a particular situation [purpose];

(B) draw conclusions and make predictions by analyzing trends in scatterplots; and

(C) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plots, line graphs, stem and leaf plots, [construct] circle graphs, bar graphs, box and whisker plots, [and] histograms, and Venn diagrams, with and without the use of technology.
(8.13) **Probability and statistics.** The student evaluates predictions and conclusions based on statistical data.

The student is expected to:
(A) evaluate methods of sampling to determine validity of an inference made from a set of data; and
(B) recognize misuses of graphical or numerical information and evaluate predictions and conclusions based on data analysis.

(8.14) **Underlying processes and mathematical tools.** The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school.

The student is expected to:
(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;
(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.

(8.15) **Underlying processes and mathematical tools.** The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models.

The student is expected to:
(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and
(B) evaluate the effectiveness of different representations to communicate ideas.

(8.16) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions.

The student is expected to:
(A) make conjectures from patterns or sets of examples and nonexamples; and
(B) validate his/her conclusions using mathematical properties and relationships.
Chapter 111. Texas Essential Knowledge and Skills for Mathematics
Subchapter C. High School

§111.31. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades 9-12.
The provisions of this subchapter shall be implemented beginning September 1, 1998, and at that
time, shall supersede §75.63(e)-(g) of this title (relating to Mathematics).

§111.32. Algebra I (One Credit).
(a) Basic understandings.
(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic
understandings of number, operation, and quantitative reasoning; patterns, relationships, and
algebraic thinking; geometry; measurement; and probability and statistics are essential
foundations for all work in high school mathematics. Students will continue to build on this
foundation as they expand their understanding through other mathematical experiences.
(2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra;
symbols provide powerful ways to represent mathematical situations and to express
generalizations. Students use symbols in a variety of ways to study relationships among
quantities.
(3) Function concepts. Functions represent the systematic dependence of one quantity on another. A
function is a fundamental mathematical concept; it expresses a special kind of relationship
between two quantities. Students use functions to determine one quantity from another, to
represent and model problem situations, and to analyze and interpret relationships.
(4) Relationship between equations and functions. Equations and inequalities arise as a way of asking
and answering questions involving functional relationships. Students work in many situations to set
up equations and inequalities and use a variety of methods to solve them.
(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in
understanding underlying relationships. Students use a variety of representations (concrete,
pictorial, numerical, symbolic, algorithmic, graphical, and verbal), tools, and technology
including, but not limited to, calculators with graphing capabilities, data collection devices,
and computers to model mathematical situations to solve meaningful problems.
(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics.
As they do mathematics, students continually use problem-solving, language and communication,
and reasoning (justification and proof) to make connections within and outside mathematics. Students also use
multiple representations, technology, applications and modeling, and numerical fluency in
problem-solving contexts.

(b) Knowledge and skills
(A.1) Foundations for functions. The student understands that a
function represents a dependence of one quantity on another and
and can be described in a variety of ways.
The student is expected to:
(A) describe independent and dependent quantities in
functional relationships;
(B) gather and record data and use data sets
to determine functional relationships between quantities;
(C) describe functional relationships for given problem
situations and write equations or inequalities to
answer questions arising from the situations;
(D) represent relationships among quantities using concrete
models, tables, graphs, diagrams, verbal descriptions,
equations, and inequalities; and
(E) interpret and make decisions, predictions, and critical
judgments from functional relationships.
(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:
(A) identify and sketch the general forms of linear \((y = x)\) and quadratic \((y = x^2)\) parent functions; \(\square\)
(B) identify the mathematical domains and ranges and determine \([\text{determines}]\) reasonable domain and range values for given situations, both continuous and discrete;
(C) interpret situations in terms of given graphs or creates situations that fit given graphs; and \(\square\)
(D) collect and organize data, make and interpret \([\text{makes and interprets}]\) scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make \([\text{models, predicts, and makes}]\) decisions and critical judgments in problem situations.

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:
(A) use symbols to represent unknowns and variables; and
(B) look for patterns and represent \([\text{represents}]\) generalizations algebraically.

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:
(A) find specific function values, simplify \([\text{simplifies}]\) polynomial expressions, transform and solve \([\text{transforms and solves}]\) equations, and factor \([\text{factors}]\) as necessary in problem situations; \(\square\)
(B) use the commutative, associative, and distributive properties to simplify algebraic expressions; and \(\square\)
(C) connect equation notation with function notation, such as \(y = x + 1\) and \(f(x) = x + 1\).

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:
(A) determine whether or not given situations can be represented by linear functions; \(\square\)
(B) determine the domain and range \([\text{values}]\) for \([\text{which}]\) linear functions in \([\text{make sense for}]\) given situations; and
(C) use, translate, and make connections \([\text{The student translates}]\) among \([\text{and uses}]\) algebraic, tabular, graphical, or verbal descriptions of linear functions.

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:
(A) develop the concept of slope as rate of change and determine \([\text{determines}]\) slopes from graphs, tables, and algebraic representations;
(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs; \(\square\)
(C) investigate, describe, and predict the effects of changes in \(m\) and \(b\) on the graph of \(y = mx + b\);
(D) graph and write \([\text{equations}]\) of lines given characteristics such as two points, a point and a slope, or a slope and \(y\)-intercept;
(E) determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and
algebraic representations;
(F) interpret and predict the effects of changing slope and y-intercept in applied situations; and
(G) relate direct variation to linear functions and solve problems involving proportional change.

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:
(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;
(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and
(C) interpret and determine the reasonableness of solutions to linear equations and inequalities.

(A.8) Linear functions. The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:
(A) analyze situations involving linear functions and formulate systems of linear equations in two unknowns to solve problems;
(B) solve systems of linear equations using concrete models, graphs, tables, and algebraic methods; and
(C) interpret and determine the reasonableness of solutions to systems of linear equations.

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:
(A) determine the domain and range for quadratic functions in given situations;
(B) investigate, describe, and predict the effects of changes in a on the graph of $y = ax^2 + c$; $y = ax^2$;
(C) investigate, describe, and predict the effects of changes in c on the graph of $y = ax^2 + c$; and $y = x^2 + c$;
(D) analyze graphs of quadratic functions and draw conclusions.

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:
(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and
(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x-intercepts) of the graph of the function.

(A.11) Quadratic and other nonlinear functions. The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student is expected to:
(A) use patterns to generate the laws of exponents and apply them in problem-solving situations;
(B) analyze data and represent situations involving inverse variation using concrete models, tables, graphs, or algebraic methods; and
(C) analyze data and represent situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.
§111.33. Algebra II (One-Half to One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students study algebraic concepts and the relationships among them to better understand the structure of algebra.

(3) Functions, equations, and their relationship. The study of functions, equations, and their relationship is central to all of mathematics. Students perceive functions and equations as means for analyzing and understanding a broad variety of relationships and as a useful tool for expressing generalizations.

(4) Relationship between algebra and geometry. Equations and functions are algebraic tools that can be used to represent geometric curves and figures; similarly, geometric figures can illustrate algebraic relationships. Students perceive the connections between algebra and geometry and use the tools of one to help solve problems in the other.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, algorithmic, graphical, and verbal), tools, and technology (including, but not limited to, powerful and accessible hand-held calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use reasoning, as well as multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(b) Knowledge and skills.

(2A.1) Foundations for functions. The student uses properties and attributes of functions and applies functions to problem situations.

(A) identify [For a variety of situations, the student identifies] the mathematical domains and ranges of functions and determine [determines] reasonable domain and range values for continuous and discrete [given] situations; and

(B) collect and organize [In solving problems, the student collects data and records results, organizes the data, make and interpret [makes] scatterplots, fit [fits] the graph of a [curves to the appropriate parent] function to the data, interpret [interprets] the results, and proceed [proceeds] to model, predict, and make decisions and critical judgments.

(2A.2) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) use tools including [matrices,] factoring [ ] and properties of exponents to simplify expressions and to transform and solve equations; and [ ]

(B) use complex numbers to describe the solutions of quadratic equations.

(C) The student connects the function notation of f(x) =
situations.

(2A.3) Foundations for functions. The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations.

The student is expected to:
(A) analyze situations and formulate systems of equations in two or more unknowns or inequalities in two or more unknowns to solve problems;
(B) use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities; and
(C) interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.

(2A.4) Algebra and geometry. The student connects algebraic and geometric representations of functions.

The student is expected to:
(A) identify and sketch graphs of parent functions, including linear \((f(x) = x)\), quadratic \((f(x) = x^2)\), square root \((y = \sqrt{x})\), inverse \((y = \frac{1}{x})\), exponential \((f(x) = a^x)\), and logarithmic \((f(x) = \log_a x)\) functions, absolute value of \(x\), square root of \(x\) \((f(x) = \sqrt{x})\), and reciprocal of \(x\) \((f(x) = 1/x)\);
(B) extend parent functions with parameters such as \(a\) in \(f(x) = ax\) and describe the effects of the parameter changes on the graph of parent functions; and
(C) describe and analyze the relationship between a function and its inverse [The student recognizes inverse relationships between various functions].

(2A.5) Algebra and geometry. The student knows the relationship between the geometric and algebraic descriptions of conic sections.

The student is expected to:
(A) describe a conic section as the intersection of a plane and a cone;
(B) in order to sketch graphs of conic sections to relate simple parameter changes in the equation to corresponding changes in the graph;
(C) identify symmetries from graphs of conic sections;
(D) identify the conic section from a given equation; and
(E) use the method of completing the square.

(2A.6) Quadratic and square root functions. The student understands that quadratic functions can be represented in different ways and translates among their various representations.

The student is expected to:
(A) determine the reasonable domain and range values of quadratic functions, as well as interpret and determine the reasonableness of solutions to quadratic equations and inequalities;
(B) relate representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions; and
(C) determine a quadratic function from its roots or a graph.

(2A.7) Quadratic and square root functions. The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations.

The student is expected to:
(A) use characteristics of the quadratic parent function to sketch the related graphs and connect between the \(y = ax^2 + bx + c\) and the \(y = a(x - h) + k\) symbolic representations of quadratic functions; and
(B) use the parent function to investigate, describe, and predict the effects of changes in \(a\), \(h\), and \(k\) on the graphs of \(y = a(x - h) + k\) form of a function in applied
(2A.8) **Quadratic and square root functions.** The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving quadratic functions and formulate quadratic equations or inequalities to solve problems.

(B) analyze and interpret the solutions of quadratic equations using discriminants and solve quadratic equations using the quadratic formula.

(C) compare and translate between algebraic and graphical solutions of quadratic equations and.

(D) solve quadratic equations and inequalities using graphs, tables, and algebraic methods.

(2A.9) **Quadratic and square root functions.** The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) use the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describe limitations on the domains and ranges.

(B) relate representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions.

(C) determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities.

(D) determine solutions of square root equations and inequalities using graphs, tables, and algebraic methods.

(E) determine solutions of square root inequalities using graphs and tables.

(F) analyze situations modeled by square root functions, formulate equations or inequalities, select a method, and solve problems and.

(G) connect inverses of square root functions with quadratic functions using square root functions.

(2A.10) **Rational functions.** The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) use quotients of polynomials to describe the graphs of rational functions, predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic behavior.

(B) analyze various representations of rational functions with respect to problem situations.

(C) determine various representations of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities.

(D) determine the solutions of rational equations and inequalities using graphs, tables, and algebraic methods.

(E) determine solutions of rational inequalities using graphs and tables.

(F) analyze a situation modeled by a rational function, formulate an equation or
inequality composed of a linear or quadratic function, and solve the problem; and (G)

generate the student uses direct and inverse variation functions to model and make predictions in problem situations involving direct and inverse variation.

(2A.11) Exponential and logarithmic functions. The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) develop the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses; (B) use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior; (C) determine the reasonable domain and range values of exponential and logarithmic functions, as well as interpret and determine the reasonableness of solutions to exponential and logarithmic equations and inequalities; (D) determine solutions of exponential and logarithmic equations and inequalities using graphs, tables, and algebraic methods; (E) determine solutions of exponential and logarithmic inequalities using graphs and tables; and (F) analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem.

§111.34. Geometry (One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Geometric thinking and spatial reasoning. Spatial reasoning plays a critical role in geometry; geometric shapes and figures provide powerful ways to represent mathematical situations and to express generalizations about space and spatial relationships. Students use geometric thinking to understand mathematical concepts and the relationships among them.

(3) Geometric figures and their properties. Geometry consists of the study of geometric figures of zero, one, two, and three dimensions and the relationships among them. Students study properties and relationships having to do with size, shape, location, direction, and orientation of these figures.

(4) The relationship between geometry, other mathematics, and other disciplines. Geometry can be used to model and represent many mathematical and real-world situations. Students perceive the connection between geometry and the real and mathematical worlds and use geometric ideas, relationships, and properties to solve problems.

(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal [algebraic, and coordinate]), tools, and technology (including, but not limited to, powerful and accessible hand-held calculators with graphing capabilities, data collection devices, and computers [with graphing capabilities] to solve meaningful problems by representing and transforming figures transforming figures] and analyzing relationships and proving things about them).
(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, [computation in problem-solving contexts,] language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use [as well as] multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts [justification and proof].

(b) Knowledge and skills.

(G.1) Geometric structure. The student understands the structure of, and relationships within, an axiomatic system.

The student is expected to:

(A) develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems;

(B) recognize [Through] the historical development of geometric systems[and the student recognizes that] and know mathematics is developed for a variety of purposes

(C) compare and contrast the structures and implications of Euclidean and non-Euclidean geometries.

(G.2) Geometric structure. The student analyzes geometric relationships in order to make and verify conjectures.

The student is expected to:

(A) use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships; and []

(B) make [The student makes and verifies] conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

(G.3) Geometric structure. The student applies [understands the importance of] logical reasoning to justify and prove mathematical statements [justification, and proof in mathematics].

The student is expected to:

(A) determine the validity [The student determines if the converse] of a conditional statement, its converse, inverse, and contrapositive; [is true or false.]

(B) construct and justify statements about geometric figures and their properties; []

(C) use logical reasoning [The student demonstrates what it means] to prove statements are true and find counter examples to disprove [mathematically that] statements that are false; [true.]

(D) use inductive reasoning to formulate a conjecture; and []

(E) use deductive reasoning to prove a statement.

(G.4) Geometric structure. The student uses a variety of representations to describe geometric relationships and solve problems.

The student is expected to select an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.

(G.5) Geometric patterns. The student uses a variety of representations to describe geometric relationships and solve problems [identifies, analyzes, and describes patterns that emerge from two- and three-dimensional geometric figures].

The student is expected to:

(A) use numeric and geometric patterns to develop algebraic expressions representing geometric properties; [to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles.]

(B) use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and
solids, and angle relationships in polygons and circles;
(C) use properties of transformations and their compositions
to make connections between mathematics and the real
world, [in applications] such as tessellations; and [or
fractals.]
(D) identify and apply [The student identifies and applies]
patterns from right triangles to solve meaningful
problems, including special right triangles (45-45-90 and
30-60-90) and triangles whose sides are Pythagorean
triples.

(G.6) Dimensionality and the
geometry of location. The
student analyzes the relationship
between three-dimensional
geometric figures [objects] and
related two-dimensional
representations and uses these
representations to solve problems.

The student is expected to:
(A) describe and draw the intersection of a given plane with
various [The student describes, and draws cross
sections and other slices of] three-dimensional
geometric figures [objects.]
(B) use [The student uses] nets to represent and construct
three-dimensional geometric figures; and [objects.]
(C) use orthographic and isometric views [The student uses
top, front, side, and corner views] of three-dimensional
geometric figures [objects] to represent and construct
three-dimensional geometric figures [create accurate
and complete representations] and solve problems.

(G.7) Dimensionality and the
geometry of location. The
student understands that
coordinate systems provide
convenient and efficient ways of
representing geometric figures
and uses them accordingly.

The student is expected to:
(A) use one- and two-dimensional coordinate systems to
represent points, lines, rays, line segments, and figures;
(B) use slopes and equations of lines to investigate
geometric relationships, including parallel lines,
perpendicular lines, and special segments of triangles
and other polygons; and [•]
(C) derive and use [The student develops and uses]
formulas involving length, slope, [including distance] and
midpoint.

(G.8) Congruence and the geometry
of size. The student uses tools to
determine measurements of
geometric figures and extends
measurement concepts to find
perimeter, area, [perimeter,] and
volume in problem situations.

The student is expected to:
(A) find areas of regular polygons, circles, and composite
figures;
(B) find areas of sectors and arc lengths of circles using
proportional reasoning; [•]
(C) derive, extend, and use [The student develops, extends,
and uses] the Pythagorean Theorem; and [•]
(D) find surface areas and volumes of prisms, pyramids,
spheres, cones, [and] cylinders, and composites of these
figures in problem situations.

(G.9) Congruence and the geometry
of size. The student analyzes
properties and describes
relationships in geometric figures.

The student is expected to:
(A) formulate and conjectures about the properties of
parallel and perpendicular lines based on explorations
and concrete models; [•]
(B) formulate and test conjectures about the properties and
attributes of polygons and their component parts based
on explorations and concrete models; [•]
(C) formulate and test conjectures about the properties and
attributes of circles and the lines that intersect them
based on explorations and concrete models; and [•]
(D) analyze the characteristics of polyhedra and other three-
(G.10) **Congruence and the geometry of size.** The student applies the concept of congruence to justify properties of figures and solve problems.

The student is expected to:

(A) use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane; and

(B) justify and apply triangle congruence relationships.

(G.11) **Similarity and the geometry of shape.** The student applies the concepts of similarity to justify properties of figures and solve problems.

The student is expected to:

(A) use and extend similarity properties and transformations to explore and justify conjectures about geometric figures;

(B) use ratios to solve problems involving similar figures;

(C) develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods; and

(D) describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems.

§111.35. **Precalculus (One-Half to One Credit).**

(a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998, and at that time shall supersede §75.63(bb) of this title (relating to Mathematics). Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisites: Algebra II, Geometry.

(b) Introduction.

(1) In Precalculus, students continue to build on the K-8, Algebra I, Algebra II, and Geometry foundations as they expand their understanding through other mathematical experiences. Students use symbolic reasoning and analytical methods to represent mathematical situations, to express generalizations, and to study mathematical concepts and the relationships among them. Students use functions, equations, and limits as useful tools for expressing generalizations and as means for analyzing and understanding a broad variety of mathematical relationships. Students also use functions as well as symbolic reasoning to represent and connect ideas in geometry, probability, statistics, trigonometry, and calculus and to model physical situations. Students use a variety of representations (concrete, pictorial, numerical, symbolic, algorithmic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model functions and equations and solve real-life problems.

(2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, justification and proof, and numerical fluency/computation in problem-solving contexts.

(c) Knowledge and skills.

(P.1) The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic, trigonometric, and piecewise-
defined functions.

(D) recognize and use connections among significant values [points] of a function (zeros, [roots], maximum values [points], [and] minimum values, etc. [points]), points on the graph of a function, and the symbolic representation of a function; and

(E) investigate the concepts of continuity, end behavior, [vertical and horizontal] asymptotes, and limits and connect these characteristics to functions represented graphically and numerically [the graph of a function].

(P.2) The student interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems [within a context].

The student is expected to:

(A) apply basic transformations, including \( a \cdot f(x), f(x) + d, f(x - c), f(b \cdot x) \), and compositions with absolute value functions, including \( |f(x)| \), and \( f(|x|) \), to the parent functions;

(B) perform operations including composition on functions, find inverses, and describe these procedures and results verbally, numerically, symbolically, and graphically; and

(C) investigate identities graphically and verify them symbolically, including logarithmic properties, trigonometric identities, and exponential properties.

(P.3) The student uses functions and their properties, tools and technology, to model and solve meaningful [real-life] problems.

The student is expected to:

(A) investigate properties of trigonometric and polynomial functions;

(B) use functions such as logarithmic, exponential, trigonometric, polynomial, etc. to model real-life data;

(C) use regression to determine the appropriateness of a linear function to model real-life data (including using technology to determine the correlation coefficient);

(D) use properties of functions to analyze and solve problems and make predictions; and

(E) solve problems from physical situations using trigonometry, including the use of Law of Sines, Law of Cosines, and area formulas and incorporate radian measure where needed.

(P.4) The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems.

The student is expected to:

(A) represent patterns using arithmetic and geometric sequences and series;

(B) use arithmetic, geometric, and other sequences and series to solve real-life problems;

(C) describe limits of sequences and apply their properties to investigate convergent and divergent series; and

(D) apply sequences and series to solve problems including sums and binomial expansion.

(P.5) The student uses conic sections, their properties, and parametric representations, as well as tools and technology, to model physical situations.

The student is expected to:

(A) use conic sections to model motion, such as the graph of velocity vs. position of a pendulum and motions of planets;

(B) use properties of conic sections to describe physical phenomena such as the reflective properties of light and sound;

(C) convert between parametric and rectangular forms of functions and equations to graph them; and
The student uses vectors to model physical situations.

The student is expected to:
(A) use the concept of vectors to model situations defined by magnitude and direction; and
(B) analyze and solve vector problems generated by real-life situations.

§111.36. Mathematical Models with Applications (One-Half to One Credit).
(a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Algebra I.
(b) Introduction.
(1) In Mathematical Models with Applications, students continue to build on the K-8 and Algebra I foundations as they expand their understanding through other mathematical experiences. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, to model information, and to solve problems from various disciplines. Students use mathematical methods to model and solve real-life applied problems involving money, data, chance, patterns, music, design, and science. Students use mathematical models from algebra, geometry, probability, and statistics and connections among these to solve problems from a wide variety of advanced applications in both mathematical and nonmathematical situations. Students use a variety of representations (concrete, pictorial, numerical, symbolic, algorithmic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to link modeling techniques and purely mathematical concepts and to solve applied problems.
(2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, justification and proof, and numerical fluency [computation] in problem-solving contexts.
(c) Knowledge and skills.
(M.1) The student uses a variety of strategies and approaches to solve both routine and non-routine problems.
The student is expected to:
(A) compare and analyze various methods for solving a real-life problem;
(B) use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines; and
(C) select a method to solve a problem, defend the method, and justify the reasonableness of the results.
(M.2) The student uses graphical and numerical techniques to study patterns and analyze data.
The student is expected to:
(A) interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, [and] scatterplots, line plots, stem and leaf plots, and box and whisker plots to draw conclusions from the data;
(B) analyze numerical data using measures of central tendency, variability, and correlation in order to make inferences;
(C) analyze graphs from journals, newspapers, and other sources to determine the validity of stated arguments; and
(D) use regression methods available through technology to describe various models for data such as linear, quadratic, exponential, etc., select the most appropriate model, and use the model to interpret information.
(M.3) The student develops and
implements a plan for collecting and analyzing data in order to make decisions.

(A) formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions;

(B) communicate methods used, analyses [analysis] conducted, and conclusions drawn for a data-analysis project by written report, visual display, oral report, or multi-media presentation; and

(C) determine the appropriateness of a model for making predictions from a given set of data.

(M.4) The student uses probability models to describe everyday situations involving chance.

The student is expected to:

(A) compare theoretical and empirical probability; and

(B) use experiments to determine the reasonableness of a theoretical model such as binomial, geometric, etc.

(M.5) The student uses functional relationships to solve problems related to personal income.

The student is expected to:

(A) use rates, linear functions, and direct variation to solve problems involving personal finance and budgeting, including compensations and deductions;

(B) solve problems involving personal taxes; and

(C) analyze data to make decisions about banking.

(M.6) The student uses algebraic formulas, graphs, and amortization models to solve problems involving credit

The student is expected to:

(A) analyze methods of payment available in retail purchasing and compare relative advantages and disadvantages of each option;

(B) use amortization models to investigate home financing and compare buying and renting a home; and

(C) use amortization models to investigate automobile financing and compare buying and leasing a vehicle.

(M.7) The student uses algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning.

The student is expected to:

(A) analyze types of savings options involving simple and compound interest and compare relative advantages of these options;

(B) analyze and compare coverage options and rates in insurance; and

(C) investigate and compare investment options including stocks, bonds, annuities, and retirement plans.

(M.8) The student uses algebraic and geometric models to describe situations and solve problems.

The student is expected to:

(A) use geometric models available through technology to model growth and decay in areas such as population, biology, and ecology;

(B) use trigonometric ratios and functions available through technology to calculate distances and model periodic motion; and

(C) use direct and inverse variation to describe physical laws such as Hook's, Newton's, and Boyle's laws.

(M.9) The student uses algebraic and geometric models to represent patterns and structures.

The student is expected to:

(A) use geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and architecture; and

(B) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music.
TEKS High School Significant Changes

This document is intended as a resource for the trainer and not necessarily to be copied for participants. However, different trainers have different styles, thus the document is formatted so that if it is copied on a black and white copier, the changes are still apparent. New refinements are in **bold red and underlined**, deletions are in **plain blue text with a strike through**.

**TEKS Algebra I Significant Changes**

(a) Basic understandings.

  (3) Function concepts. Functions represent the systematic dependence of one quantity on another. Students use functions to represent and model problem situations and to analyze and interpret relationships.

  (3) Function concepts. A function is a fundamental mathematical concept; it expresses a special kind of relationship between two quantities. Students use functions to determine one quantity from another, to represent and model problem situations, and to analyze and interpret relationships. **The change here is that there is to be greater emphasis on the relationship – developing the ability to find either quantity from the other, rather than a one-directional dependence of one quantity on another. A complete understanding of the concept of a function necessitates an understanding of the interdependence of the two variables.**

  (4) Relationship between equations and functions. Equations arise as a way of asking and answering questions involving functional relationships. Students work in many situations to set up equations and use a variety of methods to solve these equations.

  (4) Relationship between equations and functions. Equations and inequalities arise as a way of asking and answering questions involving functional relationships. Students work in many situations to set up equations and inequalities and use a variety of methods to solve them. **The significant change is in the addition of inequalities. Inequalities are generalizations of equations, and students need to be able to make connections between equations, inequalities, and the situations that they model.**

  (5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, numerical, algorithmic, and graphical), tools, and technology, including, but not limited to, powerful and accessible hand-held calculators and computers with graphing capabilities and model mathematical situations to solve meaningful problems.

  (5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with
**graphing capabilities, data collection devices, and computers** to model mathematical situations to solve meaningful problems. This rewording more explicitly requires teachers to emphasize and demonstrate concepts in a variety of ways to help students build a more complete understanding of functions and equations. The ultimate purpose is that students not only improve their understanding of these important concepts, but also their communication of their ideas and questions. Problem solving requires verbal and written communication of ideas about what is being learned, not just manipulation of symbols. More emphasis should be placed on conceptual understanding, not just computations.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, **computation in problem-solving contexts**, language and communication, connections within and outside mathematics, and reasoning, as well as multiple representations, applications and modeling, and justification and proof. The change here (besides rewording for clarity) is the addition of technology and numerical fluency. The emphasis is on numerical fluency, rather than just computation. From the elementary school level and upwards, students are expected to develop numerical fluency, and that development is continued throughout the high school courses as well. Students need to be able to apply their knowledge in new contexts, not just work problems similar to examples they have seen.

(b) Knowledge and skills.
(A.1) **Foundations for functions.** The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.
   (E) The student interprets and **makes inferences** from functional relationships. The student is expected to:
   (E) interpret and **make decisions, predictions, and critical judgments** from functional relationships. The change here is subtle. Inference involves drawing conclusions from given facts or data; making decisions involves acting upon such a conclusion, prediction involves application of such a conclusion to a similar situation, and critical judgments involves the evaluation of such a conclusion. In each case, the student is expected go beyond the inference and build upon it for deeper application and evaluation.

(A.2) **Foundations for functions.** The student uses the properties and attributes of functions.
(B) For a variety of situations, the student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

The student is expected to:
(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete.

The addition of continuous and discrete is designed to emphasize that these are distinctly different situations, and students should be able to distinguish between them, see the relationship between them, and understand the differences in the situations they model. The development of this concept is continued in Algebra II. (See TEK 2A.1(A).)

(D) In solving problems, the student collects and organizes data, makes and interprets scatterplots, and models, predicts, and makes decisions and critical judgments.

The student is expected to:
(D) collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.

To enrich the study of functions, the TEKS call for the inclusion of problem situations which can illustrate how mathematics can model aspects of the" real world." (In real life, functions arise from data gathered through observations or experiments). A discussion of correlation is also very useful in helping students interpret scatterplots, and in giving them appropriate terminology with which to do so. This is an important foundation on which students will build their understanding in Algebra II.

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:
(C) connect equation notation with function notation, such as \( y = x + 1 \) and \( f(x) = x + 1 \).

Function notation is introduced in Algebra I now instead of Algebra II. This gives students more time to become familiar with the notation for functions, which is a fundamental concept of Algebra I.

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

(C) The student translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

The student is expected to:
(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

The addition of making connections among the different representations of linear functions is extremely important. The tendency is to show the different representations without making any clear connections between them; as a result, students have disjointed ideas of a function that have no connection or relationship, and they cannot move easily from one representation to another. Connecting the representations, and showing how they are all different pictures of the same object (a function!), teachers can help students build a much deeper, more complete understanding of functions.

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

(Note that the change here filters through all the objectives, (A) through (G), so that the language of zeros of functions and intercepts of graphs is emphasized throughout.)

(E) The student determines the intercepts of linear functions from graphs, tables, and algebraic representations.

The student is expected to:

(E) determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representations.

The change here is similar to that of (A.5) (C). Connecting the zeros and the intercepts also reinforces the connection of a linear equation to a linear function, and the graphical, equation, and tabular representations of that function, and gives students terminology with which to talk about these relationships.

(A.8) Linear functions. The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

(A) The student analyzes situations and formulates systems of linear equations to solve problems.

The student is expected to:

(A) analyze situations and formulate systems of linear equations in two unknowns to solve problems.

This is a clarification; in Algebra I, students are expected to learn how to solve systems of two unknowns, while systems of three unknowns are addressed in Algebra II. (See TEK 2A.3 (A).)

(A.9) Quadratic and other non-linear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

(B) The student investigates, describes, and predicts the effects of changes in a on the graph of \( y = ax^2 \).

The student is expected to:
(B) investigate, describe, and predict the effects of changes in $a$ on the graph of $y = ax^2 + c$.

(C) The student investigates, describes, and predicts the effects of changes in $c$ on the graph of $y = x^2 + c$.

The student is expected to:
(C) investigate, describe, and predict the effects of changes in $c$ on the graph of $y = ax^2 + c$.

Both of these changes are a generalization of changing parameters of quadratic functions. The tendency for most teachers is to isolate parameter changes and teach them individually without showing the combined effect of more than one change. After demonstrating the individual effects of single parameter changes, teachers should then help students “put it all together.”

(A.10) Quadratic and other non-linear functions. The student understands there is more than one way to solve a quadratic equation and solve them using appropriate methods.

(B) The student relates the solutions of quadratic equations to the roots of their functions.

The student is expected to:
(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related function, and the horizontal intercepts ($x$-intercepts) of the graph of the function.

This change is another that reinforces making connections between the different representations of a function. Connecting the different representations, and giving the language with which to communicate the connections, is vital to helping students master the concept of a function.
(a) Basic understandings.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, numerical, algorithmic, and graphical), tools, and technology, including, but not limited to, powerful and accessible hand-held calculators and computers with graphing capabilities and model mathematical situations to solve meaningful problems.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems. (See the explanation under Algebra I changes.)

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, computation in problem-solving contexts, language and communication, connections within and outside mathematics, and reasoning, as well as multiple representations, applications and modeling, and justification and proof.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

There are several important changes here. Students need to be able to make their own connections, forming for themselves the framework that they will use as the basis of their understanding and application of new concepts. The addition of technology and numerical fluency means that students need to have opportunities to use technology in meaningful ways and in activities that will help them to develop an understanding of what methods are appropriate for use in solving problems with which they have little or no prior experience.

(b) Knowledge and skills.

(2A.1) Foundations for functions. The student uses properties and attributes of functions and applies functions to problem situations.

(A) For a variety of situations, the student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

The student is expected to:
(A) identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations.

The change here is the clarification and distinction between continuous and discrete situations. This was first introduced in Algebra I (see TEK A.2 (B)), and further developed in Algebra II. Since this concept is continued here, student mastery of the concept is not expected in Algebra I; however, teachers should not have to introduce the vocabulary of continuous and discrete in Algebra II as there is an expectation that students will already have an introductory level knowledge of this concept.

(B) In solving problems, the student collects data and records results, organizes the data, makes scatterplots, fits the curves to the appropriate parent function, interprets the results, and proceeds to model, predict, and make decisions and critical judgments.

The student is expected to:
(B) collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

The additions of interpretation and graph-fitting build upon what was supposed to have been learned in Algebra I. As students collect and organize data, they will begin to see how real-world situations can be modeled as functions, and they will be able to meaningfully interpret their data. The correlation coefficient can be taught in this context, giving students a measurement of the relationship between variables. In addition, graphing calculators can be a tremendous aid in determining lines of best fit; teachers can also use them to demonstrate non-linear curves of best fit that would be difficult or tedious to calculate by hand.

(2A.2) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.
(A) Matrices removed
(C) Moved to algebra I

(2A.3) Foundations for functions. The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations.
(A) The student analyzes situations and formulates systems of equations or inequalities in two or more unknowns to solve problems.

The student is expected to:
(A) analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems;

In Algebra I, students are expected to learn how to form and solve systems of equations in two unknowns. This is extended in Algebra II in two different directions: solving systems of three unknowns, and solving a system of inequalities (still with two unknowns). (See TEKS A.8 (A).)
(2A.4) Algebra and geometry. The student connects algebraic and geometric representations of functions.
(A) The student identifies and sketches graphs of parent functions, including linear \((y = x)\), quadratic \((y = x^2)\), square root \((y = \sqrt{x})\), inverse \((y = 1/x)\), exponential \((y = a^x)\), and logarithmic \((y = \log_a x)\) functions.
(The student is expected to:
(A) identify and sketch graphs of parent functions, including linear \((f(x) = x)\), quadratic \((f(x) = x^2)\), exponential \((f(x) = a^x)\), and logarithmic \((f(x) = \log_a x)\) functions, \textbf{absolute value of} \(x\) \((f(x) = |x|)\), square root of \(x\) \((f(x) = \sqrt{x})\), and \textbf{reciprocal of} \(x\) \((f(x) = 1/x)\).

Here, the “inverse function” is appropriately renamed the reciprocal function (as “inverse function” has a much more generalized definition). Also, function notation is used, since it was already introduced in Algebra I, and students would be expected to use and understand it in Algebra II.

(B) The student extends parent functions with parameters such as \(m\) in \(y = mx\) and describes parameter changes on the graph of parent functions.
(The student is expected to:
(B) extend parent functions with parameters such as \(a\) in \(f(x) = a/x\) and describe the effects of the parameter changes on the graph of parent functions.

The change here is intended to emphasize the use of both linear and non-linear functions. Teachers may have been limiting their examples to only linear functions, and they need to show that parameter changes affect non-linear functions and linear functions in the same way. Students also need to be able to describe these changes, both verbally and in written form.

(C) The student recognizes inverse relationships between various functions.
(The student is expected to:
(C) describe and analyze the relationship between a function and its inverse.

Students should be able to describe and analyze relationships between functions and their inverses, rather than just recognizing that there is a relationship. This means that teachers need to teach students the language of inverse relationships so that students can describe them in both oral and written forms; teachers must also help students to understand what inverses do, how they work, when they exist, and how to find them.

(2A.8) Quadratic and square root functions. The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.
(D) The student solves quadratic equations and inequalities.
(The student is expected to:
(D) solve quadratic equations and inequalities \textbf{using graphs, tables, and algebraic methods}.}
Problem solving should not be limited to only symbolic manipulation; students should learn how to solve equations and inequalities in other ways that may be faster or less prone to error. They should also be able to interpret their solutions in context, apply them in different contexts, and evaluate when a solution must be excluded. All of these representations can help students to understand what they are doing, rather than just manipulating symbols. (See the multiple representations emphasis in Basic understandings statement (5).)

(2A.9), (2A.10), (2A.11) (D) and (E) all separate solving inequalities from equations and place it in a new objective, eliminating the specification of using algebraic methods to solve inequalities.

Solutions to inequalities often require non-algebraic methods; thus solutions to equations and inequalities have been separated.

(2A.9) Quadratic and square root functions. The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

(G) The student expresses inverses of quadratic functions using square root functions.

The student is expected to:

(G) connect inverses of square root functions with quadratic functions.

Since inverses of quadratic functions are not functions, care must be taken in any discussion of inverses of quadratic functions and connecting them to square root functions. The emphasis has been reversed so that students are led to see that while there is an inverse relationship between quadratic and square-root functions, they are not directly inverses of each other. A discussion of domain and range is vital to helping students understand this inverse relationship.

(2A.10) Rational functions. The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

(A) The student uses quotients to describe the graphs of rational functions, describes limitations on the domains and ranges, and examines asymptotic behavior.

The student is expected to:

(A) use quotients of polynomials to describe the graphs of rational functions, predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic behavior.

The first change here is just a clarification that reinforces the definition of rational functions as quotients of polynomial functions. The second change is somewhat ambiguous in meaning. This group has interpreted this change to mean the effects of the parameters a, b, and c in the basic rational function \( f(x) = \frac{x}{x-b} + c \). By understanding how these parameter changes affect the graph, students will gain a better understanding of domains, ranges, end behavior, and asymptotic behavior of rational functions.
(a) Basic understandings.

(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, algebraic, and coordinate), tools, and technology, including, but not limited to, powerful and accessible hand-held calculators and computers with graphing capabilities to solve meaningful problems by representing figures, transforming figures, analyzing relationships, and proving things about them.

(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to solve meaningful problems by representing and transforming figures and analyzing relationships.

The major changes here focus on using multiple representations that include the use of technology. Note that data collection devices are a requirement now, so schools will need to provide classroom sets for teachers and students to use in explorations. Proofs are not explicitly included here; however, they are implied in the analysis of relationships among figures and in the transformations of figures.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, computation in problem-solving contexts, language and communication, connections within and outside mathematics, and reasoning, as well as multiple representations, applications and modeling, and justification and proof.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts.

The change here (besides rewording for clarity) is the addition of technology and numerical fluency. The emphasis is on numerical fluency, rather than just computation. From the elementary school level and upwards, students are expected to develop numerical fluency, and that development is continued throughout the high school courses as well. Students need to be able to apply their knowledge in new contexts, not just work problems similar to examples they have seen.

(b) Knowledge and skills.

(G.3) Geometric structure: The student understands the importance of logical reasoning, justification, and proof in mathematics.

(G.3) Geometric structure: The student applies logical reasoning to justify and prove mathematical statements.
The major change here is the change from understanding to application; note that this applies to all of the sub-topics, objectives (A) through (E). This requires more of the student. To apply logical reasoning, they must understand its importance and act on it.

(A) The student determines if the converse of a conditional statement is true or false.

The student is expected to:

(A) determine the validity of a conditional statement, its converse, inverse, and contrapositive.

This change is from only teaching the converse to including also the inverse and the contrapositive. This adds a dimension to the logical reasoning that was not previously present by examining a statement for truth from several different perspectives.

(C) The student demonstrates what it means to prove mathematically that statements are true.

The student is expected to:

(C) use logical reasoning to prove statements are true and find counterexamples to disprove statements that are false.

This is a clarification of what proofs are designed to do. Not only should students be able to logically show that something is true; they also need to be able to prove that something is NOT true, and develop techniques to do this. i.e. How do you show that a conditional statement is false? This also begins to develop the concept of proof by contradiction.

(G.5) Geometric patterns: The student identifies, analyzes, and describes patterns that emerge from two and three dimensional geometric figures.

(G.5) Geometric patterns. The student uses a variety of representations to describe geometric relationships and solve problems.

As in Algebra I and II, the emphasis is on multiple representations. The more ways a student can think about and describe geometric relationships, the deeper his/her understanding of those concepts will be; in addition, he/she will be more likely to be able to apply learned concepts to solving problems in new contexts.

(A) The student is expected to use numeric and geometric patterns to develop algebraic expressions representing geometric properties.

This is an addition not in the previous TEKS. (Note that the former (A) is now in (B).) Students need to be able to use the patterns they observe to actually predict and describe the relationships they see in symbolic form, making connections between geometric relationships and their algebraic expression. Students also need to be able to determine when a geometric pattern can be represented algebraically. This continues the idea of multiple representations that is emphasized throughout Algebra I and Algebra II.

(D) The student identifies and applies patterns from right triangles to solve problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.
The student is expected to:

(D) **identify and apply** patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.  
The emphasis is on meaningful applications. Students often ask, “When am I going to use this?” By showing them practical applications, they will see the necessity for geometry and be more motivated (we would hope!) to learn it.

(G.6) **Dimensionality and the geometry of location:** The student analyzes the relationship between three-dimensional objects and related two-dimensional representations and uses these representations to solve problems.

This change is better vocabulary.

(A) The student describes, and draws cross sections and other slices of three-dimensional objects;  
The student is expected to:  
(A) **describe and draw the intersection of a given plane with various three-dimensional geometric figures**;  
This change is better vocabulary.

(C) The student uses top, front, side, and corner views of three-dimensional objects to create accurate and complete representations and solve problems.  
The student is expected to:  
(C) **use orthographic and isometric views of three-dimensional geometric figures to represent and construct three-dimensional geometric figures and solve problems**.  
This is an update on vocabulary, as well as an emphasis on using special drawings to help to visualize geometric figures.

(G.7) **Dimensionality and the geometry of location.** The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly.  
(C) The student develops and uses formulas including distance and midpoint.  
The student is expected to:  
(C) **derive and use formulas involving length, slope, and midpoint**.  
This change removes “distance” and replaces it with “length” and adds slope. This is more general; rather than limiting the idea to just distances on the coordinate plane, students are expected to connect distances with lengths (not necessarily on the plane) and be able to use the technique of finding distance on the coordinate plane to find lengths in other contexts. In addition, slope is an important concept of Algebra I, so there is connection to linear functions, their applications, and their graphical
representations, reinforcing again the multiple representations that are emphasized in both Algebra I and Algebra II.

**G.8 Congruence and the geometry of size:** The student extends measurement concepts to find area, perimeter, and volume in problem situations.

**G.8 Congruence and the geometry of size.** The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations.

The addition of the use of tools of measurement is significant in several ways. First, students should recognize what can be used as a tool of measurement, and how it is used; this includes traditional tools, such as a ruler or a protractor (what is the significance of the various marks on a ruler and on a protractor?), but also non-traditional tools, such as cubes for measuring volume, a piece of string for measuring length, etc. Second, students must understand what attribute is being measured with these tools (we do not measure a desk, but its length and width), and what measurements are involved in calculating perimeter, area, and volume. Third, students need to be very familiar with the different units of measure and the conversions between them. And lastly, students need to understand the limitations of these tools; students need to recognize that all measurements are approximate, and that the accuracy of the measurement depends upon the accuracy of the tool. Note that this skill is first introduced in the elementary and middle school grades; thus mastery is not yet assumed, although students should have considerable experience by this point.

(A) The student finds areas of regular polygons and composite figures.

The student is expected to:

(A) find areas of regular polygons, circles, and composite figures.

This refinement adds finding the areas of circles as well as other two-dimensional geometric figures. While this is not an obvious change, it is important in that it emphasizes that students need to be able to understand and use formulas that involve irrational numbers, not just integers or rational numbers.

(D) The student finds surface areas and volumes of prisms, pyramids, spheres, cones, and cylinders in problem situations.

The student is expected to:

(D) find surface areas and volumes of prisms, pyramids, spheres, cones, cylinders, and composites of these figures in problem situations.

Students need to see how composite figures can be broken apart and different formulas used to find areas or volumes of the parts to make up the whole. (How can you find the equation that can be used to find the volume of a composite figure?)

(G.9) Congruence and the geometry of size. The student analyzes properties and describes relationships in geometric figures.

(D) The student analyzes the characteristics of three-dimensional figures and their component parts.

The student is expected to:
(D) analyze the characteristics of polyhedra and other three-dimensional figures and their component parts based on explorations and concrete models.

The addition of polyhedra is an emphasis on new vocabulary. Students need to learn appropriate vocabulary with which to describe and communicate their thoughts and questions about three-dimensional figures both verbally and in written form. The addition of “based on explorations and concrete models” continues from the previous TEKS the pattern of using explorations and concrete models to give meaning and relevance to all of these concepts.

(G.10) Congruence and the geometry of size. The student applies the concept of congruence to justify properties of figures and solve problems.

(A) The student uses congruence transformations to make conjectures and justify properties of geometric figures.

The student is expected to:

(A) use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane.

The addition here gives yet another reinforcement to multiple representations. A figure on a coordinate plane can give another connection between graphs and tables and formulas. (How can coordinate geometry be used to show two figures on a coordinate plane are congruent?)

(G.11) Similarity and the geometry of shape. The student applies the concepts of similarity to justify properties of figures and solve problems.

(A) The student uses similarity properties and transformations to explore and justify conjectures about geometric figures.

The student is expected to:

(A) use and extend similarity properties and transformations to explore and justify conjectures about geometric figures.

The addition of “and extend” is important; students are expected to extend the properties and transformations of similarity, not just use them. Teachers must therefore provide opportunities for extension in ways that are meaningful to students.

(D) The student describes the effect on perimeter, area, and volume when length, width, or height of a three-dimensional solid is changed and applies this idea in solving problems.

The student is expected to:

(D) describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems.

This change is a generalization of the effects of dimension changes; instead of focusing only on three-dimensional figures, this change makes this objective apply to all geometric figures. (What is the effect on perimeter, area and volume when dimensions of similar figures are changed?)
<table>
<thead>
<tr>
<th>TEKS</th>
<th>New Content</th>
<th>More Specific</th>
<th>Deleted</th>
<th>Moved</th>
<th>Change in Math Vocabulary</th>
<th>Reworded (no change in content)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Algebra I</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(a) (3)</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Emphasizes interrelationship of the quantities that define a function; emphasizes the relationship between the quantities.</td>
</tr>
<tr>
<td>(a) (4)</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Added inequalities</td>
</tr>
<tr>
<td>(a) (5)</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td>Emphasizes the use of multiple representations of a function</td>
</tr>
<tr>
<td>(a) (6)</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td>Addition of technology and numerical fluency</td>
</tr>
<tr>
<td>A.1 (E)</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>Student is expected to do more than just make a judgment; he/she is expected to use it to make decisions and extend to other situations that are similar.</td>
</tr>
<tr>
<td>A.2 (B)</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Emphasizes that continuous and discrete situations should both be given and distinguished as different.</td>
</tr>
<tr>
<td>A.2 (D)</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>A study of correlation has been added</td>
</tr>
<tr>
<td>A.4 (C)</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Function notation is now introduced in Algebra I instead of Algebra II</td>
</tr>
<tr>
<td>A.5 (C)</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Making connections is specified as part of the expectation of using multiple representations of functions</td>
</tr>
<tr>
<td>A.6 (E)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td>Adds the term “zero of a linear function” to the context of intercepts</td>
</tr>
<tr>
<td>A.8 (A)</td>
<td></td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td>Clarifies that only systems with two unknowns are to be addressed at this level</td>
</tr>
<tr>
<td>A.9 (B) &amp; (C)</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Emphasizes that parameter changes should not be taught exclusively in isolation</td>
</tr>
<tr>
<td>A.10 (B)</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Vocabulary is given for zeros and intercepts (“root” is inappropriate vocabulary for use in functions; it should be used in the context of equations, not functions)</td>
</tr>
<tr>
<td><strong>Algebra II</strong></td>
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<tr>
<td>(a) (5)</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>Emphasizes the use of multiple representations of a function</td>
</tr>
<tr>
<td>(a) (6)</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Addition of technology and numerical fluency</td>
</tr>
<tr>
<td>2A.1 (A)</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Emphasizes that continuous and discrete situations should both be given and distinguished as different.</td>
</tr>
<tr>
<td>TEKS</td>
<td>New Content</td>
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<tr>
<td>2A.1 (B)</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td>Interpretation of scatterplots added; better language used to describe fitting a graph to data</td>
<td></td>
</tr>
<tr>
<td>2A.2 (A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Matrices is removed</td>
<td></td>
</tr>
<tr>
<td>2A.2 (C)</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td>Moved to algebra I (see A.4 (C))</td>
<td></td>
</tr>
<tr>
<td>2A.3 (A)</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Clarifies that only two unknowns will be investigated in systems of inequalities</td>
<td></td>
</tr>
<tr>
<td>2A.4 (A)</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>Absolute value added; better language used: inverse is removed and reciprocal used to describe the function f(x)=1/x</td>
<td></td>
</tr>
<tr>
<td>2A.4 (B)</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Emphasizes the use of both linear and non-linear examples in investigating parameter changes.</td>
<td></td>
</tr>
<tr>
<td>2A.4 (C)</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td>Emphasizes more than just recognition needed; students need to describe and analyze.</td>
<td></td>
</tr>
<tr>
<td>2A.8 (D)</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Specifies that multiple solution methods should be taught, not just symbolic manipulation</td>
<td></td>
</tr>
<tr>
<td>2A.9 (D) &amp; (E) 2A.10 2A.11</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>All these K&amp;S separate solving inequalities from equations, eliminating the specification of using algebraic methods to solve inequalities (which often do not exist).</td>
<td></td>
</tr>
<tr>
<td>2A.9 (G)</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Inverses of quadratic functions are not functions; this refinement helps teachers to see and teach this important distinction between square root functions and the inverses of quadratic functions.</td>
<td></td>
</tr>
<tr>
<td>2A.10 (A)</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>Clarification of quotients that are rational functions; adds predictions of the effects of parameter changes (see document for more details)</td>
<td></td>
</tr>
</tbody>
</table>

**Geometry**

<p>| (a) (5) | | X | X | | | Emphasizes the use of multiple representations of a function |
| (a) (6) | X | | | | X | Addition of technology and numerical fluency |
| G.3 (A) | X | | | X | | Application, not just understanding |
| G.3 (C) | | | | X | | Clarification of what proofs are designed to do |
| G.5 | X | | | | X | Emphasizes multiple representations |
| G.5 (A) | X | | | | | New material in (A), representing a geometric pattern algebraically; former (A) moved to (B) |</p>
<table>
<thead>
<tr>
<th>TEKS</th>
<th>New Content</th>
<th>More Specific</th>
<th>Deleted</th>
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<th>Change in Math Vocabulary</th>
<th>Reworked (no change in content)</th>
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<tbody>
<tr>
<td>G.5 (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td>Emphasizes that problems should be meaningful; otherwise, just reworded for clarity</td>
</tr>
<tr>
<td>G.6</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>“Objects” replaced with “geometric figures”</td>
</tr>
<tr>
<td>G.6 (A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td>Better vocabulary</td>
</tr>
<tr>
<td>G.6 (C)</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td>Updated vocabulary; using special drawings emphasized</td>
</tr>
<tr>
<td>G.7 (C)</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td>“distance” is replaced with “length” and “slope”, not just limiting distances to the plane</td>
</tr>
<tr>
<td>G.8</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>Using tools is added</td>
</tr>
<tr>
<td>G.8 (A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td>Circles added to finding areas</td>
</tr>
<tr>
<td>G.8 (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Composite figures added</td>
</tr>
<tr>
<td>G.9 (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td>New vocabulary: polyhedra; emphasizes explorations and concrete models</td>
</tr>
<tr>
<td>G.10 (A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Emphasizes using figures on a coordinate plane, and reinforces the use of multiple representations.</td>
</tr>
<tr>
<td>G.11 (A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Extension added</td>
</tr>
<tr>
<td>G.11 (D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td>Generalization of the effects of dimension changes; not limited to three-dimensional figures; and more than one parameter may be changed</td>
</tr>
</tbody>
</table>